## How Many Squares? <br> Teacher Notes

## Introduction

The aim of this activity is for students to realise the importance of a systematic approach when searching for all the solutions to a particular problem. It gives teachers the opportunity to discuss what it means to be systematic and to convey that there may be several equally good systematic approaches. The context for the activity is working on a geoboard - how many different squares can be found? - how will we know when we have found them all? The activity is divided into two parts
a) Areas (How many different sized squares are there?)
b) Places (In how many positions can each square be placed?)


## Resources

The TI-Nspire document, geoboards.tns, by Andy Kemp, provides pages with square and circular geoboards. The tns file HowManySquares.tns is a simple adaptation of that file. Students will also need square dot paper, or the worksheets provided.
Although the document is perfectly usable by students working alone, it was particularly designed for use with TI-Nspire Navigator with the Screen Capture tool.

## Skills required

Students need to be able to move from one page of the document to another by pressing either
(tr) or ©fris.

To simulate an elastic band making shapes on the geoboard they need to use the polygon tool,

> menu (9) Shapes (4) Polygon

Once the shape is made, grab a corner point to move to a new position:. place the cursor over the point, press (atr) and notice the closed hand. Then drag it to a new position and press 娄.
Students also need to be able to measure the area using the measurement tool
(menu) (8) Measurement (2) Area.
They will need to use the slider to change the number of points on the grid: click on the up or down arrow. The biggest possible grid has 15 rows of 15 points.

## a). How many squares with different areas can be made?

Start the activity with a 25-peg board and show students how to draw a 2 by 2 square on page 1.4 of the tns file. Record the square on square dot paper and write it its area in the middle. Then show them how to move points on the grid like an elastic band and ask them to make another square with a different area. Using screen capture different squares can be identified and recorded. When the first student makes a tilted square they could be made "presenter" to show the others how to make this square. Students should easily be able to work out the area of each square by counting gridsquares or you may prefer to show them the measurement tool to find this area

How many more squares are there? How can we be sure we have them all? Students could record their results on square dot paper. There is also a worksheet provided which could be used with lower ability students.

| 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 |  |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 |  |



| square name | area |
| :---: | :---: |
| 1 by 1 | 1 |
| up 1 along 1 | 2 |
| 2 by 2 | 4 |
| up 1 along 2 | 5 |
| up 2 along 2 | 8 |
| 3 by 3 | 9 |
| up 1 along 3 | 10 |
| 4 by 4 | 16 |



On a 25-peg grid there are 8 squares with different areas
Students could record the areas of the squares that they find on the spreadsheet page 1.5
As they find a new squarearea they enter it in column $B$ after entering squarename in column $A$.


Looking at the row number tells them how many squareareas they have recorded. They can easily sort the areas they have recorded into ascending order (teacher stressing systematic recording) and, using Navigator, the teacher can quickly see their recorded results from looking at the spreadsheet page. Column $C$ is used for the second part of the activity.

Once the problem has been completed, ask students to use the slider to make a 64-peg grid and find all the squares on this grid, recording their squares in a systematic way.



| square name | area |
| :---: | :---: |
| 1 by 1 | 1 |
| up 1 along 1 | 2 |
| 2 by 2 | 4 |
| up 1 along 2 | 5 |
| up 2 along 2 | 8 |
| 3 by 3 | 9 |
| up 1 along 3 | 10 |
| up 2 along 3 | 13 |
| 4 by 4 | 16 |
| up 1 along 4 | 17 |
| up 3 along 3 | 18 |
| up 2 along 4 | 20 |
| 5 by 5 | 25 |
| or <br> up 3 along 4 |  |
| up 1 along 5 | 26 |
| up 2 along 5 | 29 |
| up 1 along 6 | 36 |
| 7 by 7 | 37 |
| 49 |  |

On a 64-peg grid there are 18 squares with different areas

## b). How many different ways can squares be placed on a grid

Start the activity by asking the students to use the slider to make a 16 -peg grid and draw a 2 by 2 square. Discuss in how many distinct places this sized square can be positioned.
Record the places on square dot paper. Using Navigator's screen capture facility the different placements can be identified and recorded. What about squares of other sizes?
Start by recording the positions of the non-tilted squares first, then move on to tilted ones, perhaps using the measurement tool to find the area. Navigator allows students to be made "Presenter" to show the class the different placements of a square.
Did the student start with the square in the best place for systematic listing?
How many more ways of placing squares are there? How can we be sure we have them all?


$\cdots$
:
-


| square name | different places |
| :---: | :---: |
| 1 by 1 | 9 |
| 2 by 2 | 4 |
| 3 by 3 | 1 |
| up 1 along 1 | 4 |
| up 1 along 2 | 1 |
| up 2 along 1 | 1 |
| total | 20 |

## On a 16-peg grid there are $\mathbf{2 0}$ different places

Once the problem has been completed, ask students to use the slider to make a 25 -peg grid and find all the ways of placing squares on this grid, recording their results in a systematic way.


Again students could record their work in the spreadsheet page 1.5 .

To delete the previous data, place the cursor in the column heading, up arrow to hightlight the column, and press menu) (3) Data (4) Clear Data



| square name | different <br> places |
| :---: | :---: |
| 1 by 1 | 16 |
| 2 by 2 | 9 |
| 3 by 3 | 4 |
| 4 by 4 | 1 |
| up 1 along 1 | 9 |
| up 1 along 2 | 4 |
| up 1 along 3 | 1 |
| up 2 along 1 | 4 |
| up 2 along 2 | 1 |
| up 3 along 1 | 1 |
| total | 50 |

## On a 25 -peg grid there are $\mathbf{5 0}$ different places

Once the problem has been completed, ask students to find all the squares on a 64-peg grid, but not by drawing all the squares. recording where the top right corner and bottom left corner can move to, makes it possible to work out the different positions efficiently using the worksheet.


| square name | different places |
| :--- | :---: |
| up 1 along 3 | $4 \times 4=16$ |



Places $=7 \times 7$

Places $=3 \times 3$


Places $=5 \times 5$

Places $=1 \times 1$

Places $=1 \times 1$

Places $=3 \times 3$



Places $=6 \times 6$


Places $=2 \times 2$


Places $=4 \times 4$

Places $=5 \times 5$


Places $=4 \times 4$

Places $=2 \times 2$



Places $=5 \times 5$


Places $=1 \times 1$


Places $=3 \times 3$


Places $=4 \times 4$


Places $=4 \times 4$


Places $=3 \times 3$


Places $=1 \times 1$


Places $=3 \times 3$


Places $=2 \times 2$


Places $=2 \times 2$


Places $=1 \times 1$


Places $=2 \times 2$


Places $=1 \times 1$


Places $=1 \times 1$

| square name | different <br> places |
| :---: | :---: |
| 1 by 1 | 49 |
| 2 by 2 | 36 |
| 3 by 3 | 25 |
| 4 by 4 | 16 |
| 5 by 5 | 9 |
| 6 by 6 | 4 |
| 7 by 7 | 1 |
| up 1 along 1 | 36 |
| up 1 along 2 | 25 |
| up 1 along 3 | 16 |
| up 1 along 4 | 9 |
| up 1 along 5 | 4 |
| up 1 along 6 | 1 |
| up 2 along 1 | 25 |
| up 2 along 2 | 16 |
| up 2 along 3 | 9 |
| up 2 along 4 | 4 |
| up 2 along 5 | 1 |
| up 3 along 1 | 16 |
| up 3 along 2 | 9 |
| up 3 along 3 | 4 |
| up 3 along 4 | 1 |
| up 4 along 1 | 9 |
| up 4 along 2 | 4 |
| up 4 along 3 | 1 |
| up 5 along 1 | 4 |
| up 5 along 2 | 1 |
| up 6 along 1 | 1 |
| total | 336 |

On a 64-peg grid there are $\mathbf{3 3 6}$ different places

