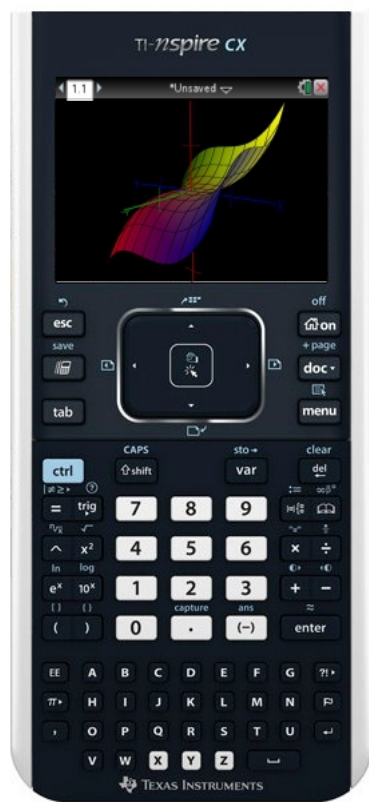


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for GCSE  
and IGCSE

David Getling

## Preface

I have been thinking of writing a book like this for quite a while, but never really had the time to do so. Then in June 2012 Bavarian International School reneged on their contract with me. Trusting in what I thought to be a reputable school, I had passed over other job opportunities, handed in the notice to my landlord in Milan, and arranged for my chattels to be moved to Munich. I was up that proverbial creek without a paddle, so returning to the UK family home seemed the most practical solution. From the incessant whining about the shortage of maths teacher in the UK you would have thought that I would have very little difficulty finding a job here. I can teach any high school area of maths, and students, who generally seem to enjoy being taught by me tell, me I do a damn good job of it – I can also teach chemistry and physics (both experiencing a shortage of teachers). Unfortunately, in the UK, and probably quite a few other places, the main selection criteria for teachers seem to be that they are cheap, young, and easily *manageable*. So I now have time to plug a gap that I feel badly needs to be filled. While I may not be able to help a school full of students on a face-to-face basis, perhaps this book will make up for the short-sightedness of so many schools.

Although, during my time abroad, I have experienced various degrees of dissatisfaction in regard to the use of graphics display calculators (GDCs), I am truly appalled by what I have seen in the UK. Never mind GCSE, even at A-level I have come across schools where GDCs are not used: in fact, there is an independent one like this just up the road from me. There are several reasons for this. Some teachers, because they haven't done their homework, believe that they are not allowed in exams – total rubbish! Many teachers are technophobic, or they are too lazy to themselves learn about these calculators, and therefore put themselves in the position of being able to appreciate their benefits. Then there are the pig-headed teachers, who take it upon themselves to decide that because students have managed for many years without these devices they should continue to do so. Even when students are introduced to GDCs the level of instruction is usually lamentable.

With a very strong technological background, I hope that, for those many unfortunate students, my book will go some way towards putting things right.

I have decided to make this book freely available, so that as many people as possible will benefit from it. However, the small-mindedness of so many schools has deprived me of my livelihood (and many students of a good maths education), so any donations will be most gratefully accepted – don't just leave it to others, otherwise I'll end up with nothing. To make a donation please visit my website at [www.getling.org/books](http://www.getling.org/books). You may also download further copies of this book from here.

Although I am making this freely available, I retain full copyright to it. You are free to print it out and are encouraged to pass on copies (printed or electronic). However, you may not sell this book, alter it in any way (including deleting **any part** of it), or produce derivative works from its content.

I hope this book helps you get a better grade in your maths exams.

David Getling

September 2013

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# 1. Introduction

## 1.1 Who is this book for?

When people think about using a graphics display calculator (GDC) they usually think of sixth form students and above. After all, a simple scientific calculator is easier to use, much cheaper, and will do all the calculations required of more junior students. This is a bit like saying that an axe and hand saw is all that a lumberjack needs to cut down trees. These may suffice, but he would probably really appreciate having a chainsaw: and he will get through a lot more trees.

When I taught in New Zealand, at Christchurch Girls' High School, I saw younger students being encouraged to use the Casio fx-9750G graphics display calculator. This enabled weaker students to tackle problems they might otherwise have been unable to do, and all students were able to work faster, and more accurately. This meant that these students were likely to achieve a higher mark in their exams than those students who had not been lucky enough to have teachers who were willing to instruct them in the use of a GDC.

So this book is written with GCSE and IGCSE students in mind, as the vast majority of UK and British run schools are too luddite to equip them with a useful tool that would give them an extra edge in their exams. Of course, as maths is much the same the world over, other students of similar age will probably also find this book useful.

## 1.2 Why the TI-Nspire CX?

For very little money you can now buy a scientific calculator that will even let you input your calculations in the same form that you might write them or see them in a text book. There are also GDCs that are considerably cheaper than the TI-Nspire CX. Any of these will do all the calculations you need for GCSE or IGCSE. So, why would you want to spend more on a Nspire? Well, there are many features this calculator possesses that the others don't, or might not implement as well.

Here are a few of the Nspire's features that will make your life easier:

- The Nspire has a good help system built in. So if you forget what a function does, or the order of its arguments, you don't have to go to a manual (and, of course, you can't in an exam).
- Not only does the Nspire have a better screen than cheaper GDCs, it also has colour. So it's easier to see what's going on if you need to plot more than one graph at the same time.
- There is a built in spreadsheet.
- In a lot of ways using the Nspire is a bit like using a computer. For instance, there's a touch pad, and the screen displays a pointer. So if, as is extremely likely, you are used to using a computer, a lot of what you do on the Nspire will feel familiar to you.

## 1.3 What will I gain by using the TI-Nspire?

The Nspire can help you work a lot faster and more accurately. Say you have a list of unsorted numbers. You can enter them into the spreadsheet and sort them instantly. You can also instantly see the mean, median, mode, and much more. The Nspire can even produce a Box and Whiskers graph

of this data.

The Nspire will solve a lot of equations for you: simple linear equations, simultaneous equations, and quadratic equations. Sure, in most exams, you probably have to show your working, but you can quickly check if you got it right. Or, knowing the right answer, might help you produce the right working.

Naturally, as per its GDC description, the Nspire can help you with a lot of your graph work. Sketching a graph and highlighting its important features is now a doddle.

## 1.4 What this book is not

Texas Instruments provides lots of free information that clearly explains how to use this calculator, and if that's not enough for you there are several comprehensive books that are easily available. This book's purpose is to show you how to use your Nspire to make your exams easier, and improve your grades. It is not a manual for teaching you the basics of your TI-Nspire.

## 1.5 How to get the best out of this book

The first thing to do is get familiar with your Nspire by learning how to use its basic features. So read the material that came with the calculator, and visit their website at [nspiringlearning.org.uk](http://nspiringlearning.org.uk) for more information. Do try the examples that you are shown, to make sure you haven't misunderstood what you read or watched. You did get the same results, didn't you?

Now, pay attention to this! Because initially using a Nspire (or any GDC) can seem more trouble than a simple calculator, I've seen students use their simple one, only reaching for their GDC when the simple calculator won't cut the mustard. This is a really bad strategy! You want using your Nspire to become second nature, so use it all the time. You will soon find that you can do most things you need to without any effort. Learning to use your mobile phone is probably harder, but it's likely that you now use it all the time without giving a second thought to its initial difficulty.

Try the examples in this book, to be sure you understand them, and then try and apply what you've learnt to similar problems in class or for homework. As has been said, maths is not a spectator sport, and neither is learning how to use your Nspire.

## 1.6 My school doesn't want me to use the TI-Nspire

This section shouldn't be necessary, but it is. First, let's clear up a misconception that too many teachers seem to have. You most certainly can use most GDCs in an exam, and this definitely includes the TI-Nspire CX. What you cannot use, in most exams, is a calculator that has a computer algebra system (CAS). CAS calculators will do things like factorizing  $x^2 - y^2$  into  $(x+y)(x-y)$ . So the **TI-Nspire CX is allowed**, but the **TI-Nspire CAS is not allowed**. Do not buy the CAS model as you will not be able to use it in your exams.

Sometimes a teacher/school is using a particular GDC and doesn't want the hassle of teaching students how to use a different one. However, you still have the right to use whichever (exam approved) calculator you chose. The teacher may not help you with it (that's why I'm writing this

book), but he most certainly can't stop you using it. Also, don't let yourself be conned into believing that the school's choice is the best, and that you will derive no benefit from using an Nspire. I've used (and taught students to use) the Casio fx-9750G, TI-83 and TI-84. So, I know what I'm saying when I say the Nspire is better: and I'm not on a commission from Texas Instrument.

Sometimes things get nasty. Several years ago, in Christchurch, New Zealand, teachers in the maths department of Burnside High School (supposedly one of the best schools) were persecuting some of the students I tutored for using a GDC. This was despite the fact that students in other schools were being encouraged to use them, to great advantage. I suggested to one parent that she asked the school to put their objection in writing. The school backed down immediately.

The Nspire will probably help you get more marks in your exam, so don't let a school con you or bully you into giving up this advantage. This is the calculator of choice in some of the latest texts published for students who are taking the International Baccalaureate Diploma: a highly respected qualification.

## 1.7 Some Tips

My first and most important tip is to make sure your calculator has the latest operating system installed. This book is based on version 3.2, which provides lots of nice new features. Where appropriate, I make use of these features. The Texas Instruments website provides OS upgrades for free, along with instructions for installing them, and details of the new features.

Get into the habit of keeping your calculator well charged, and be aware that the batteries will run down fairly quickly even if the calculator isn't being used. It really sucks when a calculator dies in the middle of something you really need it for, and it's a disaster if it runs out of juice during an exam. You don't need a special TI charger, or to plug into a computer's USB port. You can now buy very cheap chargers that look just like a wall plug with a built in USB socket: they work fine.

There is a Press-to-Test function to disable certain features during an exam. However, it's not a case of all or nothing; there are 11 options. So you need to find out which features your exam board has stipulated must be disabled, if any, but it most certainly won't be all of them. Whether he likes it or not, it's the exam invigilator's job to know how to get this right. Make sure the school knows this, because suing them for disabling something you are entitled to use can be a bit tedious. Also, make sure that you know how to get your calculator back into its normal mode.

While knowing how to get the answers with your calculator can often save you time, prevent silly errors, and even help you out when you are not too sure about something, you should still try and make sure you are capable of managing without it. Many exams, including GCSE and IGCSE, have a non-calculator paper. Your TI-Nspire will serve you well in the other paper(s), but for this one you are on your own.

You may well look at some sections and think, no I can do this stuff no sweat. Fair enough, but it might just be worth skimming it. Perhaps I do things differently from the way your teacher showed you, or perhaps you might be able to use what you see there somewhere else.

## 2. Data

### 2.1 Mean, Median, Mode, Min, Max Q1, Q3

Without doubt, you will have to deal with some of these in your exam. Here's a typical question.

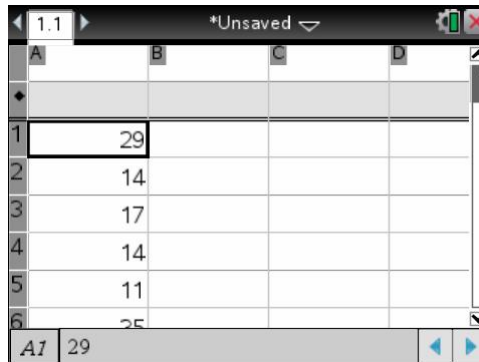
The list below shows the annual wages in thousands of pounds for employees in one of Fat Cat Enterprises regional offices.

29, 14, 17, 14, 11, 35, 12, 18, 13, 12, 15

- Find the mean, median, and mode of these wages
- Calculate the range and interquartile range of the wages.
- Why might the manager prefer to quote the mean wage when advertising a vacancy?

Let's see how the Nspire can do most of our work for us.

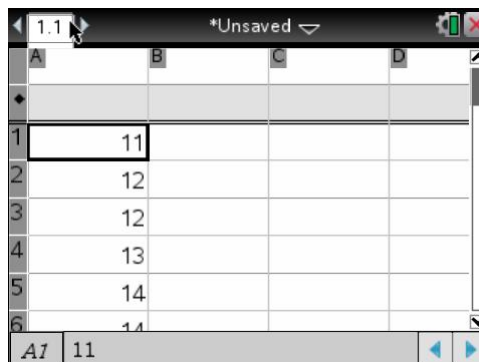
Using the spreadsheet, we enter the numbers given



A screenshot of a TI-Nspire spreadsheet window titled "1.1" and "\*Unsaved". The spreadsheet has columns A, B, C, and D, and rows 1 through 6. The data is entered in column B: row 1 is 29, row 2 is 14, row 3 is 17, row 4 is 14, row 5 is 11, and row 6 is 25. The status bar at the bottom shows "A1 29".

	A	B	C	D
1		29		
2		14		
3		17		
4		14		
5		11		
6		25		

We now select all the numbers and then press **[ctrl] [menu]**. Choose the sort option (6) and accept the defaults, to get



A screenshot of a TI-Nspire spreadsheet window titled "1.1" and "\*Unsaved". The spreadsheet has columns A, B, C, and D, and rows 1 through 6. The data is sorted in ascending order in column B: row 1 is 11, row 2 is 12, row 3 is 12, row 4 is 13, row 5 is 14, and row 6 is 14. The status bar at the bottom shows "A1 11".

	A	B	C	D
1		11		
2		12		
3		12		
4		13		
5		14		
6		14		

Apart from helping us with the mode, the only reason we need this step is to keep the examiner happy; we **need** to show him a sorted list.

As can be seen in the above picture, our first data element is already selected, which is exactly what we need. We now press **[menu]** and select **Statistics->Stat Calculations->One-Variable Statistics**

Accepting the defaults we obtain the following screen (several views).

	A	B	C	D
1			=OneVar(a[	
2	11	Title	One-Varia.	
3	12	$\bar{x}$	17.2727	
4	12	$\Sigma x$	190.	
5	13	$\Sigma x^2$	3874.	
6	14	$s_x := s_{n-1}x...$	7.69534	

	A	B	C	D
5	14	$s_x := s_{n-1}x...$	7.69534	
6	14	$\sigma_x := \sigma_n x$	7.33721	
7	15	n	11.	
8	17	MinX	11.	
9	18	$Q_1 X$	12.	
10	29	MedianX	14.	

	A	B	C	D
8	17	MinX	11.	
9	18	$Q_1 X$	12.	
10	29	MedianX	14.	
11	35	$Q_3 X$	18.	
12		MaxX	35.	
13		$SSX := \Sigma (...)$	592.182	

We've now got everything we need, but let's see how we can do this by just using a calculator window instead.

First I create a list (which I'm going to call **fat**) of numbers. I then sort it and display it, as the following screen shows.

```

fat := { 29, 14, 17, 14, 11, 35, 12, 18, 13, 12, 15 }
      { 29, 14, 17, 14, 11, 35, 12, 18, 13, 12, 15 }
SortA fat
fat { 11, 12, 12, 13, 14, 14, 15, 17, 18, 29, 35 }
  
```

I now use the **OneVar** function, followed by **stat.results**, to get exactly what I obtained with the spreadsheet.

```

fat { 11, 12, 12, 13, 14, 14, 15, 17, 18, 29, 35 }
OneVar fat
stat.results
  "Title"      "One-Variable Statistics"
  "x̄"          17.2727
  "Σx"         190.
  "Σx²"        3874.
  "s_x := s_{n-1}x" 7.69534
  "σ_x := σ_n x"  7.33721
  
```



You decide which method you prefer. At any rate, we are now ready to answer the question.

- a) For the mean we read off  $\bar{x}$ , but to keep the examiner happy we need to show

$$\text{mean} = \frac{29+14+17+14+11+35+12+18+13+12+15}{11} = 17.3$$

where I have rounded to 3s.f.

Now we copy out the sorted list, so the examiner can see where we got the median and mode

11, 12, 12, 13, 14, 14, 15, 17, 18, 29, 35

So the median is 14, as we can see on the spreadsheet, and the mode is 12 **and** 14. The calculator doesn't have a function for this, but sorting the data (which it does do) makes the mode easy to find.

- b) Reading off MaxX and MinX we get                      range = 35 – 11 = 24  
Reading off Q<sub>3</sub>X and Q<sub>1</sub>X we get                      interquartile range = 18 – 12 = 6
- c) The calculator won't help you with this. The obvious answer is that a manager would use the mean because it makes the wages look higher. When it comes to jobs, your average Joe is almost always more interested in the median wage, while the fat-cat managers like to quote the mean, because their big salaries make it look like a typical worker is better off than he really is.

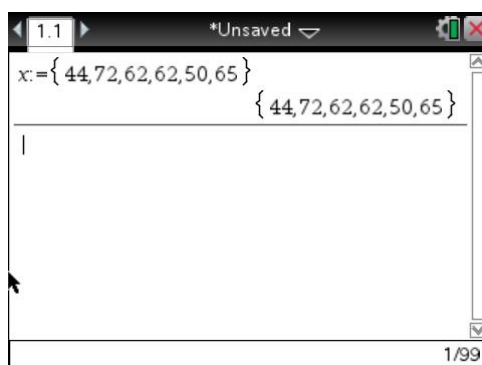
## 2.2 Box and Whiskers

Just give your Nspire a list of numbers and it will draw a box plot for you. This, of course, won't make the examiner very happy, so you need to do most of what was shown in the previous section, to show him where you got the min, max, Q1, median and Q3 from. However, often the question will give you all of these and simply ask you to draw the box plot, as in the following question.

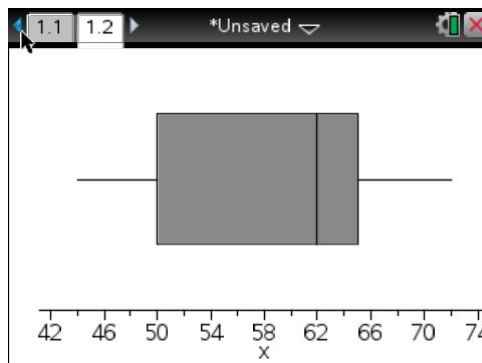
The weights of students in a class were measured and it was found that the minimum was 44kg, the maximum was 72kg, the median was 62kg, the lower quartile was 50kg, and the upper quartile was 65kg. Draw a box plot to represent this.

Simply put these numbers in a list, but **include the median twice** (can you see why you have to do this?).

We are going to use two windows for this. Our document with a calculator window, is where we enter our list.



We now add a Data & Statistics window, set the  $x$  variable to  $x$ , and choose **[menu][Plot Type][Box Plot]**.



That's it!

## 2.3 Stem and Leaf

The Nspire won't draw a stem and leaf diagram for you, but it may well be worth using its sort function to speed things up, and avoid silly mistakes. It doesn't take long to enter a list of numbers, either in a calculator app or a spreadsheet app. Then it's very easy to check that we've copied them all correctly **before** using the sortA function. Once the calculator has sorted the numbers, all the hard work has been done. It's easy to see what our stem should be, and what we are going to put in the leaves is now in order.

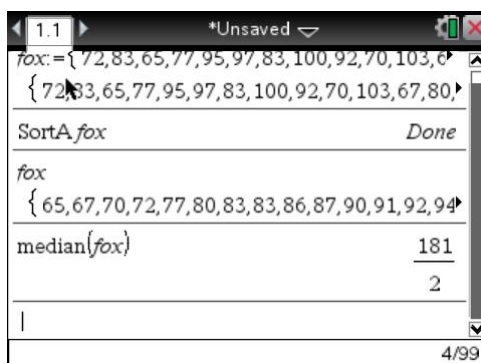
Here's a typical question.

The heights, in cm, of 22 foxglove plants are:

72, 83, 65, 77, 95, 97, 83, 100, 92, 70, 103, 67, 80, 91, 94, 102, 99, 95, 86, 90, 87, 95

- Show these heights in an ordered stem and leaf diagram.
- What is the mode of these heights?
- Find the median height.

Once this is sorted, the rest is easy. So, in a calculator document, we put all these numbers into a list, sort the list and display it. And, we might as well feed our list into the median function. This is all shown below.



- We now simply read off the sorted list to create our stem and leaf diagram

6		5 7
7		0 2 7
8		0 3 3 6 7
9		0 1 2 4 5 5 5 7 9
10		0 2 3

- It is clear, from our stem and leaf diagram, that the modal height is 95cm
- The median is  $(90+91)/2 = 90.5$ cm, which we can see shown as  $181/2$  on the calculator.

**NOTE:** If (as you should) you have made some effort to get familiar with your TI-Nspire you should know that, by default, it displays non-integers as fractions. To get a decimal representation one of the numbers in your calculation should have a decimal point in it.

## 2.4 Two Way Tables

There is probably no time saving to be made by doing these on the Nspire's spreadsheet, so you might want to skip this section. However, using the spreadsheet might slightly reduce the chance of making a silly error, and it provides a way to check your answers. Here is a question to show how we might use the spreadsheet.

Each of 300 students in a school plays one of three musical instruments: piano, violin, or flute.

Of the 174 girls, 39 play the violin.

33 boys play the flute.

196 students play the piano.

The number of students playing the violin is the same as the number of students playing the flute.

How many boys play the piano?

Let's put everything we've been told on a spreadsheet. By the way, you don't actually need the row and column labels. I've put them in to make things clearer. Notice also that I've resized the columns so that I can have more visible on the screen.

	A	B	C	D	E	F	G
1		p	v	f			
2	m			33			
3	f		39		174		
4		196			300		
5							
6							

OK, let's now use the spreadsheet to work out what's missing.

In E2 enter  $=E4-E3$

In C4 enter  $=(E4-B4)\div 2$

In D4 enter  $=C4$

In D3 enter  $=D4-D2$

In C2 enter  $=C4-C3$

In B2 enter  $=E2-D2-C2$

We now have

	A	B	C	D	E	F	G
1		p	v	f			
2	m	80	13	33	126		
3	f		39	19	174		
4		196	52	52	300		
5							
6							

We can now see our answer: it's 80.

If we want to check that everything is correct then we go to B3 and enter  $=B4-B2$ . This completes the table. We now go to an unoccupied cell, say F5, and enter  $=\text{sum}(B2:D3)$ .

Our spreadsheet now looks like

	B	C	D	E	F	G
1	p	v	f			
2	m	80	13	33	126	
3	f	116	39	19	174	
4		196	52	52	300	
5					300	
6						

Our totals, of 300, agree. So we can be fairly sure that what we've calculated is correct.

## 2.5 Stratified Sampling

Nspire's spreadsheet can do all the heavy lifting in a stratified sampling question such as the following.

This table gives information about the number of students studying foreign languages in a school.

Language	Number of male students	Number of female students
French	87	83
German	45	32
Spanish	57	61

A stratified sample of 50 students is to be taken.

- How many male students studying German should be in the sample?
- How many students studying French should be in the sample?
- How many female students should be in the sample?

Let's put this data into a spreadsheet and add row and column totals: as in the last section the labels are optional, and included just for clarity. Doing this we get

	m	f	
f	87	83	170
g	45	32	77
s	57	61	118
	189	176	365

In cell F2 we now enter  $=B2 \times 50.0 \div 365$  (notice I've written 50.0 to make things come out in decimal). We now copy cell F2 to the three rows below it. Then we copy each of these cells to the next two columns. Doing this gives us

	m	f				
f	87	83	170	11.9...	11.3...	23.2...
g	45	32	77	6.16...	4.38...	10.5...
s	57	61	118	7.80...	8.35...	16.1...
	189	176	365	25.8...	24.1...	50.

Formula bar for F2:  $= \frac{b2 \cdot 50.0}{365}$

We can now read off all the results we need

- a) Rounding 6.16, we need to include 6 male students studying German.
- b) Similarly, we should have a total of 23 students studying French.
- c) And we should have 24 female students.

## 2.6 Frequency Tables and Cumulative Frequency

There are several types of problem involving frequency tables. Three examples are given in this section.

One of the most straightforward problems would be something like the following.

The table below shows the number of McDonald's meals eaten, in one week, by pupils in a class.

Number of meals	Number of students
0	5
1	10
2	7
3	1
4	2
5	3

Work out the mean number of McDonald's meals eaten by pupils in this class.

We could, in fact, let the Nspire do all the work (or use this as a check) as follows

The screenshot shows a TI-Nspire calculator window with the following data and calculations:

$m = \{0, 1, 2, 3, 4, 5\}$	$\{0, 1, 2, 3, 4, 5\}$
$f = \{5, 10, 7, 1, 2, 3\}$	$\{5, 10, 7, 1, 2, 3\}$
$\text{mean}(m, f)$	$\frac{25}{14}$
$\frac{25}{14}$	1.78571

but just writing down 1.79 isn't going to make the examiner very happy. He wants to see a table and some totals, so we are going to use the Nspire's spreadsheet. Here are two views of what we need

The screenshot shows a TI-Nspire spreadsheet with the following data:

	m	f	mf		
1	0	5	0		
2	1	10	10		
3	2	7	14		
4	3	1	3		
5	4	2	8		
6	5	2	10		

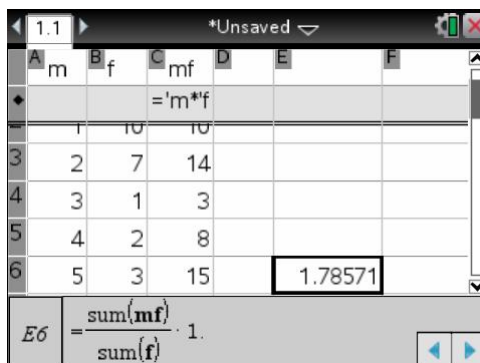
The screenshot shows a TI-Nspire spreadsheet with the following data:

	m	f	mf		
2		7	14		
4	3	1	3		
5	4	2	8		
6	5	3	15		
7		28	50		1.78571



Notice that, for a change, I've put my labels right at the top. As well as being able to see them all the time, there are other advantages to this, which I will get to in a moment. It should be clear that the first 6 rows of columns A and B are copied directly from the table we were given. In C1 I entered  $=A1 \times B1$ , and I then copied this to cells C2 to C6. In B7 I entered  $=\text{sum}(B1:B6)$ , and I then copied this to C7. Finally, to get the mean, I entered  $=C7 \div B7 \times 1.$  in E7. The “ $\times 1.$ ” was used to make the answer display as a decimal.

Now here's another way of doing things



Because the m and f were entered right at the top the Nspire treats them as list variables, whose contents are the column below them. Now, notice the “ $=m*f$ ”. This is the neat bit. It tells the spreadsheet that each element in list m is to be multiplied by the corresponding element in list f and placed in list mf, which is column C. Finally notice how I calculated the mean, which is in cell E6.

Actually, for this question you probably want the earlier spreadsheet, so that the examiner will get that nice warm fuzzy feeling when he sees your totals for the frequency and meals  $\times$  frequency.

Next, we will have a look at a table containing grouped data, and see how to answer a couple of questions based on it.

Marks m	Number of students
$0 < m \leq 20$	3
$20 < m \leq 30$	4
$30 < m \leq 40$	7
$40 < m \leq 50$	10
$50 < m \leq 60$	15
$60 < m \leq 80$	5
$80 < m \leq 100$	2

We will do two exercises with this table

- Estimate the mean mark
- Work out the cumulative frequency. In an exam this would be a prelude to drawing the cumulative frequency graph, and probably getting the quartiles from it, or something similar.

To estimate the mean we want the mid-class values. So here's how we might tackle this with a spreadsheet.

l	u	m	f	mf
1	20	10.5	3	31.5
21	30	25.5	4	102.
31	40	35.5	7	248.5
41	50	45.5	10	455.
51	60	55.5	15	822.5

l	u	m	f	mf
41	50	45.5	10	455.
51	60	55.5	15	832.5
61	80	70.5	5	352.5
81	100	90.5	2	181.
			46	2203.
				47.89...

First notice how I've entered the lower (l) and upper (u) class boundaries. Because we don't get fractions of a mark the lower values are entered as 1, 21, 31 etc. not 0, 20, 30. To get the mid-value (m) I enter  $=(A2+B2)\div 2$ . in C2. Actually, I'm using capital letters here for clarity: in fact, I enter a2 and b2, because it's quicker and case doesn't matter. C2 is then copied to C3 through to C8. The frequencies are in column D. In column E is each mid-value multiplied by its frequency. Enter  $=C2\times D2$  in cell E2 and copy the cell to E3 through to E8. Now, in D9 we enter  $=\text{sum}(D2:D8)$  and copy this cell to E9. Finally, in G9 we enter  $=E9\div D9$ , giving us a mean mark of 47.9.

Now let's have a look at calculating those cumulative frequencies. Here are images of the spreadsheet.

u	f	cf
20	3	3
30	4	7
40	7	14
50	10	24
60	15	29

u	f	cf
40	7	14
50	10	24
60	15	39
80	5	44
100	2	46

For cumulative frequency we are only interested in upper boundaries. First copy B2 to C2. Now, in C3 we enter  $=C2+B3$ . Then we copy C3 to cells C4 through to C8. And that's it! We've got the table we want to show the examiner, and we can now plot a cumulative frequency graph from it. Oh, don't forget to join the points with a smooth curve.

## 2.7 Frequency Polygon

The spreadsheet work in this section has already been covered earlier. What is new is the use of a scatter plot.

This table shows the number of hours, in a week, that students in a class spent watching TV.

Time (h hours)	Frequency
$0 \leq h < 4$	1
$4 \leq h < 8$	3
$8 \leq h < 12$	4
$12 \leq h < 16$	6
$16 \leq h < 20$	7
$20 \leq h < 24$	10
$24 \leq h < 28$	2

Draw a frequency polygon to show this.

OK, here's the spreadsheet we might use.

	lower	upper	midval	freq
1	0	4	2	1
2	4	8	6	3
3	8	12	10	4
4	12	16	14	6
5	16	20	18	7

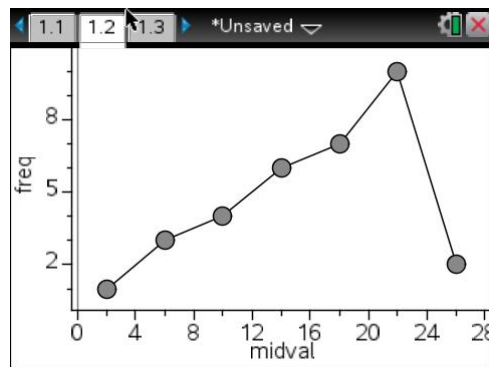
Formula bar:  $\text{midval} = \frac{\text{upper} + \text{lower}}{2}$

I've entered data in the first, second and fourth columns. Because I put my labels right at the top, they are now list variables. So, to fill the third column all I need to do is enter the formula that you can see in the cell I've left highlighted.

The most straightforward way to see what the graph should look like is to add a Data & Statistics app to my document. Then I must do three more things:

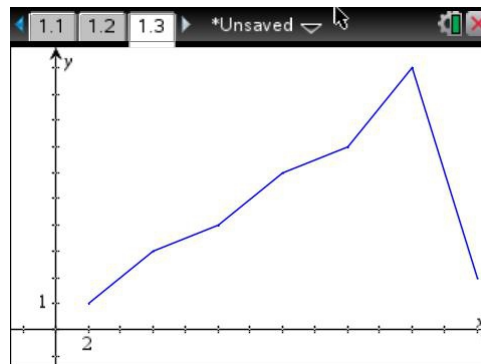
1. Choose midval as the variable for the horizontal axis. Just move the pointer near the bottom and middle of the screen and you will get a prompt.
2. Choose freq as the variable for the vertical axis. Move the pointer near to the left and about half way up.
3. After you have done these things a scatter graph will automatically be drawn. Now choose the option to join the points and you will have a fairly good idea of what your frequency polygon should look like.

Here's what your result should look like



When you copy it for the examiner you're going to draw small x's instead of circles?

Just for interest, instead of using the Data & Statistics app, I could have used the Graphs app, and chosen a plot type of Scatter Graph. This app gives me quite a few options to play with, so I have more control over what the graph looks like. Here's an example



However, the first way is quicker, so it would be the better choice in an exam.

## 2.8 Histogram

One of my reasons for writing this section is to give you a warning. The TI-Nspire does indeed have a Histogram option. However, it **will not draw the kind of histograms that will be required of you** in the exam. While the latest OS now supports variable class widths, it does not do frequency density, which is almost certainly what your exam will be testing. In fact, what the TI produces probably shouldn't be called histograms. Maybe, in a future upgrade, TI will fix this, which would be really neat.

So, no pretty pictures in this section of a TI screen showing a histogram.

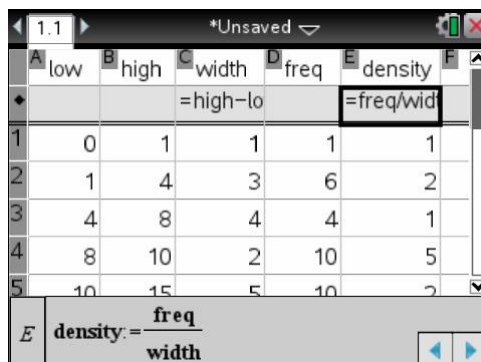
However, if you wanted to, you could use the spreadsheet to calculate your frequency densities. Here is an example that shows how to do this.

The table below shows the number of sunshine hours, per day, in July.

Sunshine (h hours)	Frequency
$0 \leq h < 1$	1
$1 \leq h < 4$	6
$4 \leq h < 8$	4
$8 \leq h < 10$	10
$10 \leq h < 15$	10

Show this on a histogram.

We could use a spreadsheet to calculate the class widths, and hence the frequency density. Here's how.



The screenshot shows a TI-Nspire spreadsheet with the following data:

	A low	B high	C width	D freq	E density
			=high-lo		=freq/width
1	0	1	1	1	1
2	1	4	3	6	2
3	4	8	4	4	1
4	8	10	2	10	5
5	10	15	5	10	2

At the bottom of the spreadsheet, the formula for density is shown as:

$$\text{density} = \frac{\text{freq}}{\text{width}}$$

If you have read the earlier sections it should be very obvious what is going on here. We can now simply read off the frequency density we need to plot for each class.

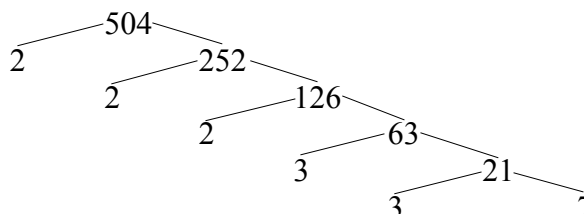
Whether it's worth doing a spreadsheet like this will depend on our data. If there's a lot, or the numbers are awkward then it might be. But, for a Y11 exam you might well be able to do most of the calculations you need in your head. In which case the only reason for doing a spreadsheet would be to reduce the chance of silly mistakes – but exam time is often very precious, with little to spare.

### 3. Number

#### 3.1 Factors, GCD and LCM

If you get these on a paper that allows you to use your calculator, the Nspire will provide you with an instant answer.

Need the prime factorisation of 504? Just enter **factor(504)** to get  $2^3 \cdot 3^2 \cdot 7$ . It's that easy, but for the examiner you would, of course, produce.



For some reason schools seem to prefer the term Highest Common Factor (HCF), whereas real mathematicians say *greatest common divisor* (gcd). Anyway, if you are asked to find the HCF of 216 and 180 simply enter **gcd(216,180)**, to get 36.

Again, for the examiner, you would probably show something like

Factors of 216	1 2 3 4 6 8 9 12 18 24 27 36 54 72 108 216
Factors of 180	1 2 3 4 5 6 9 10 12 15 18 20 30 36 45 60 90 180

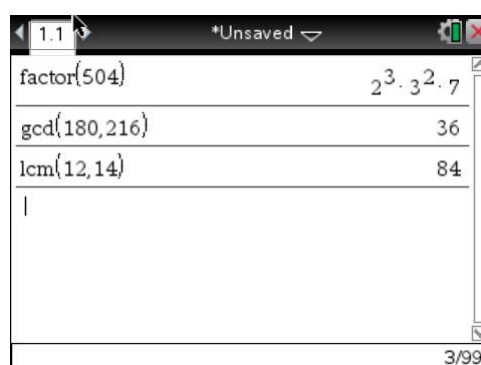
Incidentally, once again, this isn't how real mathematicians do this. They would use the Euclidean Algorithm.

If you were asked for the least common multiple of 12 and 14 typing **lcm(12,14)** would return 84. Again you would probably show something like

12	24	36	48	60	72	84
14	28	42	56	70	84	

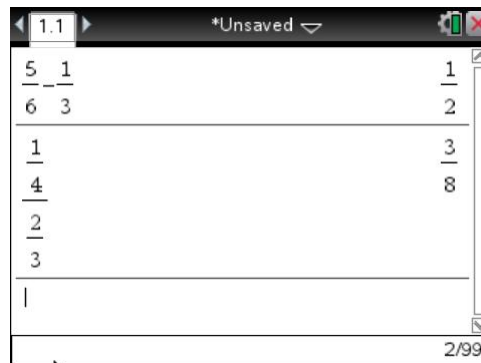
But, already knowing the answer, you wouldn't waste time writing down more multiples than you need.

Here's what all this looks like on the TI-Nspire.



## 3.2 Fractions

You just type these in as you would write them down and your Nspire will do all your sums for you. Here are a couple of examples

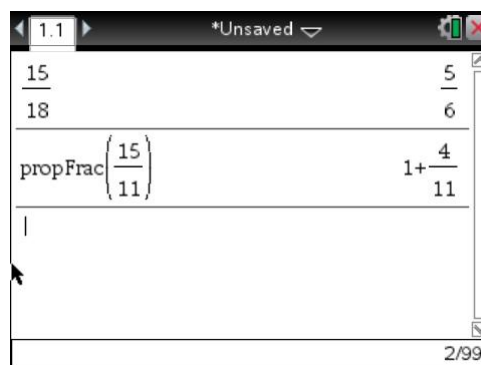


Actually, I entered the second calculation as  $\frac{1}{4} \div \frac{2}{3}$

Simplifying fractions is also very easy. Just type your fraction in and hit Enter.

And, if you want to turn an improper fraction into a mixed number just use the **propfrac** function.

Here's an example of each of these.



### 3.3 Compound Interest

Here is a typical question.

If I invest £1800 at 4% compound interest, how much will I have at the end of 3 years?

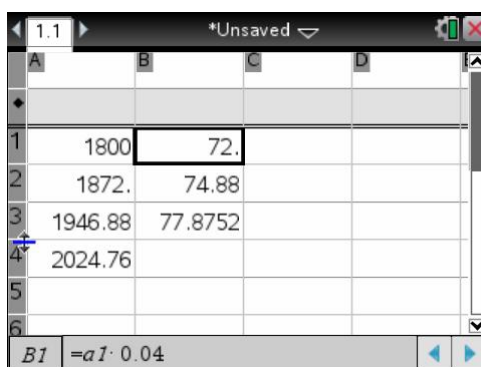
If you're a budding A or A\* student you would probably just write

$$1800 \times 1.04^3 = \text{£}2024.76$$

If you aren't happy doing it this way then the spreadsheet is your buddy.

In cell A1 enter 1800. In B1 enter  $=A1 \times 0.04$ . In A2 enter  $=A1+B1$ . Now copy A2 to A3 and A4. Then copy B1 to B2 and B3.

Here's what we get



The screenshot shows a spreadsheet window titled "1.1" and "\*Unsaved". The spreadsheet has columns A, B, C, and D, and rows 1 through 6. The data is as follows:

	A	B	C	D
1	1800	72.		
2	1872.	74.88		
3	1946.88	77.8752		
4	2024.76			
5				
6				

The formula bar at the bottom shows the formula for cell B1:  $=a1 \cdot 0.04$ .

In A1 is our investment at the beginning of the first year. In A2 is our investment at the end of the first year, which is our investment at the start of the second year. In A3 is our investment at the end of the second year, and so on. In the B column is the interest we earned each year.



### 3.4 Trial and Improvement

These questions beg us to use the Nspire's spreadsheet. Here's an examples

$x^3 - 5x + 1 = 8$  has a solution between 2 and 3. Find the value of  $x$  to 1 d.p.

Doing the same calculation again and again kind of sucks. So we let the spreadsheet do it for us.

In A1 enter 2. In A2 enter =A1+0.1, and copy this all the way down to A11. In B1 enter =A1^3-5×A1+1, and copy this all the way down to B11. This is what we get

	A	B	C	D
1	2	-1		
2	2.1	-0.239		
3	2.2	0.648		
4	2.3	1.667		
5	2.4	2.824		

Formula bar: B1 =a1^3-5\*a1+1

	A	B	C	D
6	2.5	4.125		
7	2.6	5.576		
8	2.7	7.183		
9	2.8	8.952		
10	2.9	10.889		

Formula bar: A10 =a9+0.1

Now, for a happy examiner, we write

$$2.5^3 - 5 \times 2.5 + 1 = 4.125$$

$$3^3 - 5 \times 3 + 1 = 13$$

$$2.8^3 - 5 \times 2.8 + 1 = 8.952$$

$$2.7^3 - 5 \times 2.7 + 1 = 7.183$$

Since 7.183 is closer to 8 than 8.952  $x = 2.7$  (1 d.p.)

The spreadsheet has not only saved us time, but it has significantly reduced our chances of making some simple error during all these calculations.

## 4. Algebra

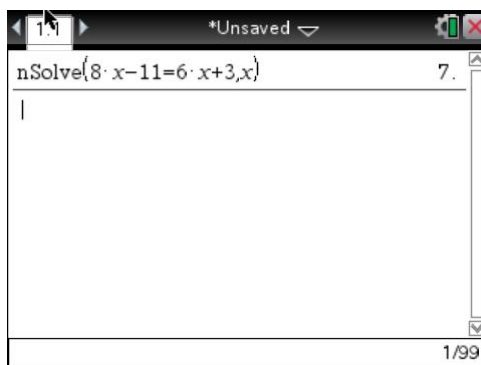
### 4.1 Linear Equations

I'm going to show you two ways to tackle these. The first, using nSolve, is very much faster. However, the graphical method allows you to see what's going on, which you might find useful in the future if you were solving something more complicated than a linear equation: when it might have several solutions.

So, here's our problem for this section.

$$\text{Solve } 8x - 11 = 6x + 3$$

And here's the quick way



Of course, to get full marks, you probably need to show something like this

$$\begin{aligned} 8x - 11 &= 6x + 3 \\ 2x - 11 &= 3 \\ 2x &= 14 \\ x &= 7 \end{aligned}$$

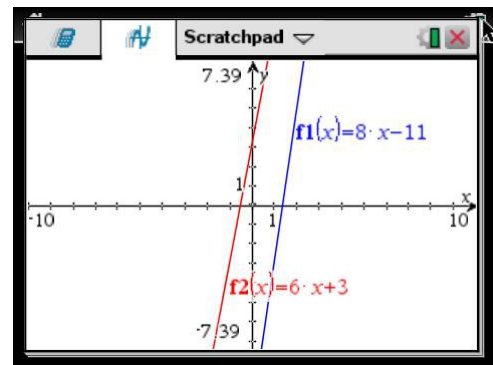
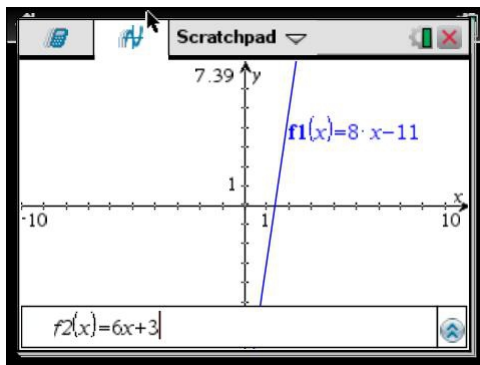
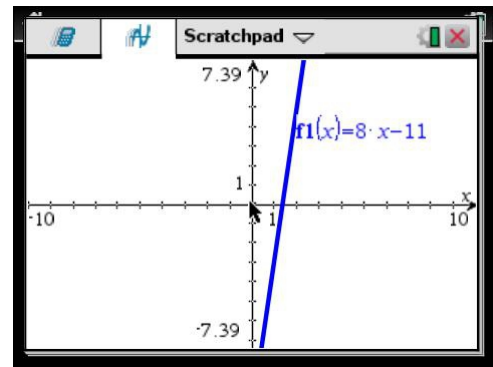
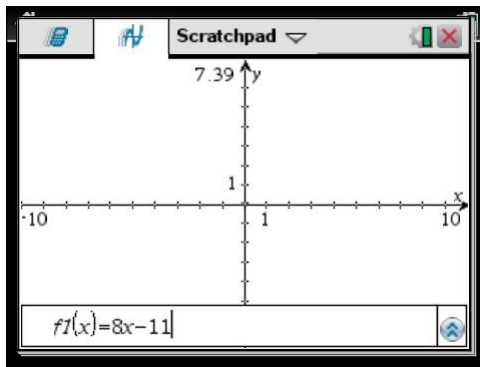
However, you know what answer you are aiming for, and this might even help you insure that your working is correct.

OK, you can skip the rest of this section if you want. We are now going to solve our problem graphically.

There are three steps:

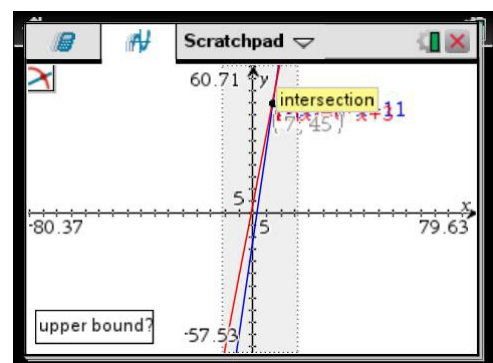
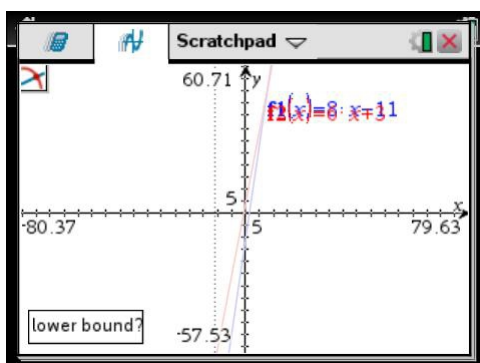
1. Plot  $f1(x) = 8x - 11$
2. Plot  $f2(x) = 6x + 3$
3. Find their intersection

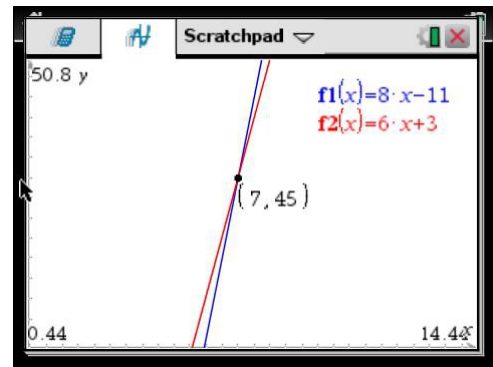
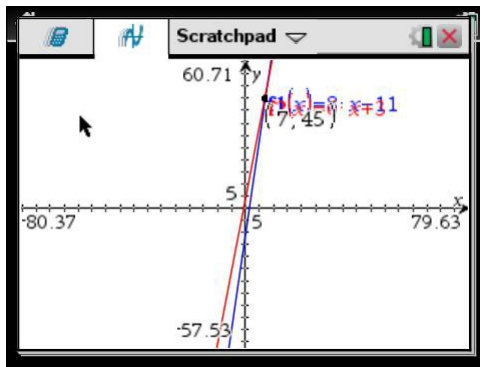
In the graph app, or the graph scratch pad, make sure the Entry Input Line is showing at the bottom by enter **[menu][View][Show Entry Line]** if it isn't visible, or **[ctrl][G]**. After the  $f1(x)=$  enter **8x - 11**. Display the Entry Input Line again, and after  $f2(x)=$  enter **6x + 3**. The four screen shots below show this.



At the moment we can't see where the lines cross, so we will use the Zoom Out option a couple of times until we can. We can also move the graph up and down, or left and right. This is all standard stuff, in your TI-Nspire documentation, that you must make yourself familiar with.

Once we can see where the lines cross we use **[menu][Analyse Graph][Intersection]**. You will now see a faint vertical line and be asked for a lower bound. So move the line to the left of where the graphs cross, and press Enter. Next, you are asked for an upper bound. This time the faint line should be to the right of where the graphs cross. Once you have done this the intersection will be marked and labelled with its coordinates. The following three screens show all this. In the fourth screen I've used the Box Zoom option and moved the labels, so that everything is a bit clearer. This example also demonstrates how nice having colour is. Although I've moved the graph labels it's still clear which line belongs to which graph.





As I already said, this is overkill for a linear equation. But, if I had to solve something like  $\sin(x) = \sin(x^2)$ , which has lots of solutions I would probably prefer to use the graphical method.

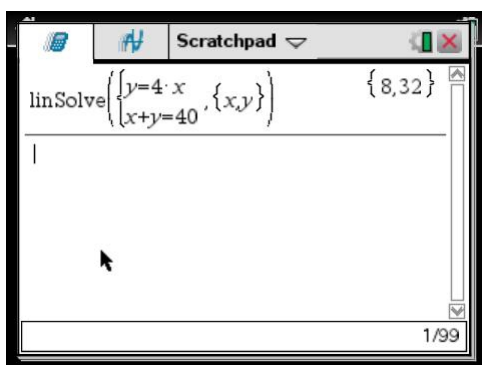
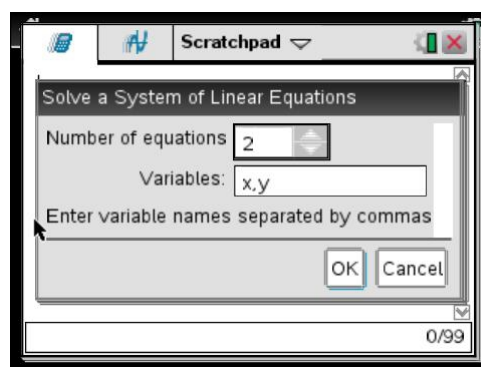
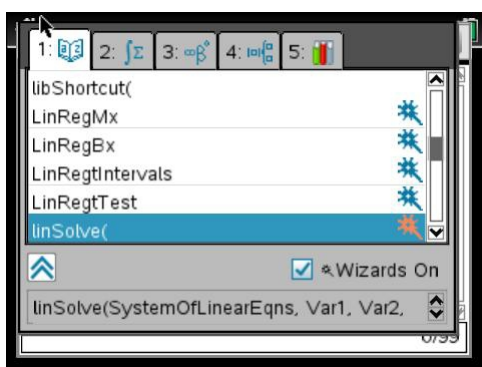
## 4.2 Simultaneous Equations

Actually we will only deal with linear simultaneous equations in this section. Your Nspire has functions specifically dedicated to solving these. We will see how to handle other, harder, simultaneous equations in the next chapter.

Compared to other GCDs that I have used, the TI-Nspire definitely has the easiest to use simultaneous equation solver. There are two ways to use it, which I'm going to demonstrate with two examples.

One way is to access the `linSolve` function from the catalogue, but make sure you have its **Wizards On** box checked. When you select this function from the catalogue, you will be asked for the number of variables you want to solve for, and their names. You will then be presented with a template in which to enter each equation.

Lets see how we do this to solve the equations  $y = 4x$  and  $x + y = 40$ .



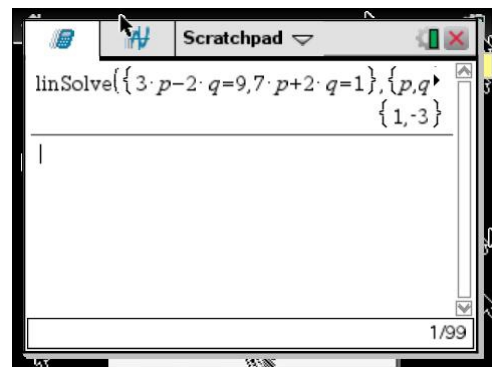
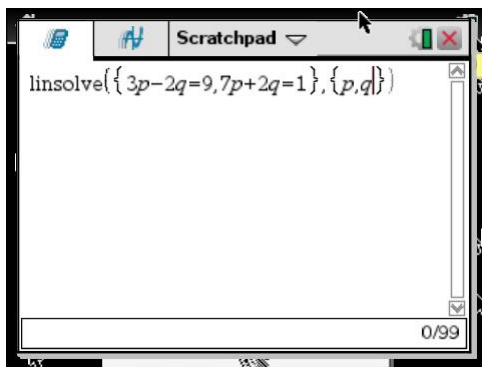
To collect full marks we would use the substitution method.

$$\begin{aligned}x + 4x &= 40 \\5x &= 40 \\x &= 8 \\y &= 4 \times 8 = 32\end{aligned}$$

Now let's have a look at another example, but we will do this one without using the catalogue and wizards. We need to give `linSolve` two arguments. The first argument is a list of the equations we wish to solve. The second argument is a list of the variables we wish to solve for.

This time our equations are  $3p - 2q = 9$  and  $7p + 2q = 1$ .

For clarity, I've shown what I've done in two screens. The first is just before I hit Enter, and the second is immediately after.



Notice that the TI-Nspire doesn't force me to use  $x$  and  $y$ . I can use any variable names I chose (as long as they satisfy what the Nspire considers to be valid – i.e. start with a letter and not a function name). I could have used *Bill* and *Ben* if I wanted to.

One question you might have is, why is the second list of variable names needed. There are two reasons. Firstly, it specifies what order you want your answers in. The second reason is that instead of numbers, like 1, 2, 3, 7 and 9 in the last two equations I could use variables that I had already assigned values to. For instance, if I'd previously entered  $a:=2$  I could have entered my equations as  $3p - aq = 9$  and  $7p + aq = 1$ . Now the calculator needs to know which two variables to solve for.

Getting back to our problem, and showing some working for it. This one cries out for the elimination method. So we would add the two equations to get

$$\begin{aligned} 10p &= 10 \\ p &= 1 \end{aligned}$$

$$\begin{aligned} 3 \times 1 - 2q &= 9 \\ -2q &= 6 \\ q &= -3 \end{aligned}$$

### 4.3 Quadratic Equations

When it comes to quadratics there are three kinds of problem you are likely to encounter:

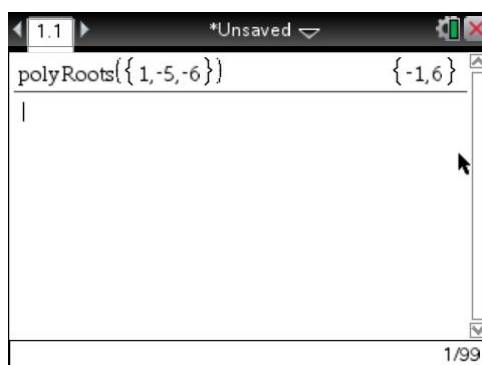
- Finding the roots of a quadratic equations
- Factorising a quadratic expression
- Completing the square of a quadratic expression

For the first of these you could either factorise first, or you could use the formula for a quadratic equation (which is given to you in the front of your exam paper). But using the polyRoots function is even easier. You have two ways of entering a quadratic into polyRoots, as the following example shows.

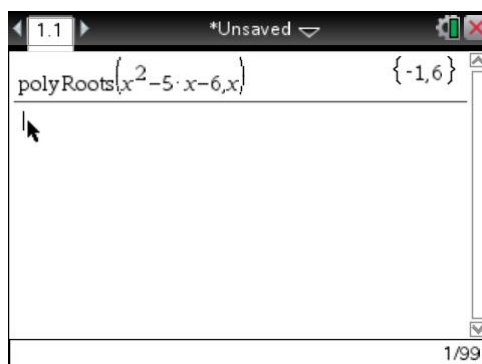
Say you were asked to solve  $x^2 = 5x + 6$ . Unless you are going to use the graphical technique that I showed you in section 4.1, you must rearrange this equation so that there is a zero on one side of it. In fact, for the first method we are going to use, the quadratic equation must be in the standard form, which in this case is

$$x^2 - 5x - 6 = 0$$

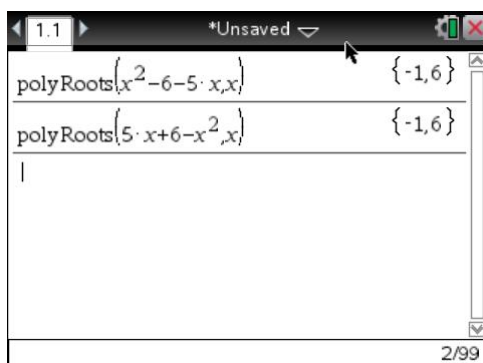
Now we enter the  $a$ ,  $b$  and  $c$  coefficients, as a list, into polyRoots. In this case  $a = 1$ ,  $b = -5$  and  $c = -6$ . This is shown below.



Alternatively, we can enter the non-zero side of the equation, and the variable we are solving for, as this screen shows.



Actually, when we do it this way polyRoots isn't too fussy about how we order things. For instance, both  $x^2 - 6 - 5x$  and  $5x + 6 - x^2$  would work, as you can see in the next screen.



Now, how can the Nspire help us factorise a quadratic expression? If the roots of a quadratic equation are  $p$  and  $q$  then it can be written as

$$k(x - p)(x - q) = 0$$

Of course, if you are just factorising an expression you are going to leave out the “=0”, aren't you? Often, for GCSE or IGCSE,  $k$  will be 1. It determines how steep a parabola that passes through  $(p,0)$  and  $(q,0)$  is: also if  $k$  is negative then the parabola is upside down.

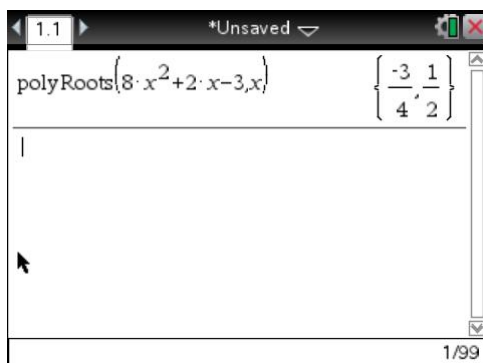
To see how all this works let's have another look at  $x^2 - 5x - 6$ . We know, from the previous example, that when this equals zero its roots are -1 and 6. Therefore its factorisation will be

$$k(x - (-1))(x - 6) = k(x + 1)(x - 6)$$

What about the  $k$ ? If we expanded this we would get a  $kx^2$  term, so  $k$  must be 1, and our final answer is

$$x^2 - 5x - 6 = (x + 1)(x - 6)$$

Let's do one more. Suppose we want to factor  $8x^2 + 2x - 3$ . Feeding this into polyRoots gives us roots of  $\frac{1}{2}$  and  $-\frac{3}{4}$ , as shown below.



So this would factor as  $k(x - \frac{1}{2})(x + \frac{3}{4})$ . If we expanded this we would get a  $kx^2$  term, which needs to be  $8x^2$ , so  $k = 8$ , and our answer is

$$8x^2 + 2x - 3 = 8(x - \frac{1}{2})(x + \frac{3}{4})$$



We can tidy this up a little. Let's multiply what's inside the first bracket by 2, and divide the 8 by 2, so everything remains the same. This gives us

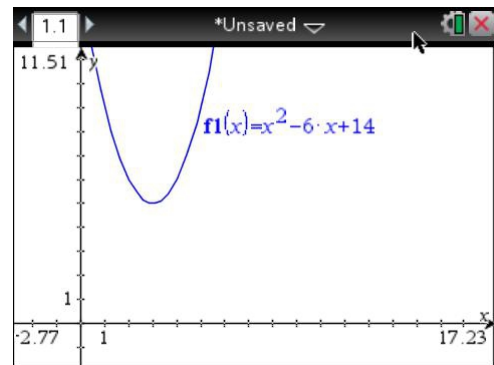
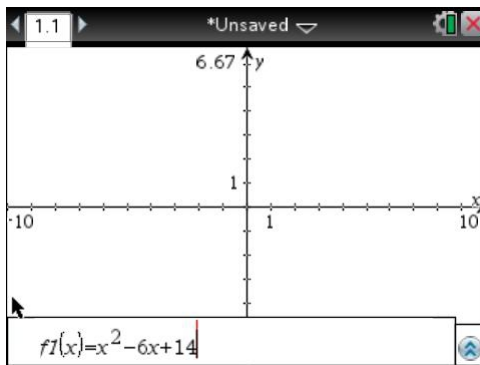
$$8x^2 + 2x - 3 = 4(2x - 1)(x + \frac{3}{4})$$

Now let's multiply what's in the second bracket by 4, and divide the 4 by 4. We finish up with

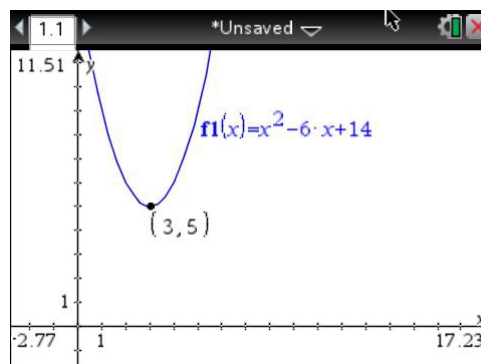
$$8x^2 + 2x - 3 = (2x - 1)(4x + 3)$$

All three answers are correct (because they are the same), but removing the fractions makes the third one look a bit neater.

Finally, in this section, we are going to see how our Nspire can help us with problems that ask us to complete the square. Say we are asked to complete the square for  $x^2 - 6x + 14$ . We use the graph app to plot this.



If the graph we see has a minimum value, as it does in this case, then we use **[menu][Analyse Graph][Minimum]**. This is what we get



Look at the coordinates of the minimum. If they were  $(p, q)$  our completed square would be

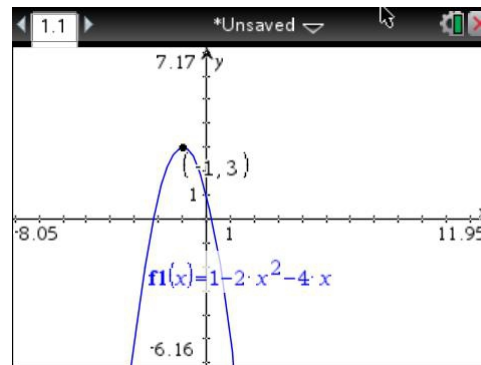
$$k(x - p)^2 + q$$

So for our example we have

$$x^2 - 6x + 14 = (x - 3)^2 + 5$$

$k = 1$ , because we only have one  $x^2$ .

If instead of a minimum the graph had a maximum we would use **[menu][Analyse Graph][Maximum]**. This is shown in the following screen for  $1 - 2x^2 - 4x$ .



So we have

$$\begin{aligned}1 - 2x^2 - 4x &= k(x - (-1))^2 + 3 \\ &= k(x + 1)^2 + 3 \\ &= -2(x + 1)^2 + 3\end{aligned}$$

or, if you prefer

$$1 - 2x^2 - 4x = 3 - 2(x + 1)^2$$

## 4.4 Arithmetic Sequences

To be honest, these are so easy most people won't need this section.

The recipe is

1. Find the difference  $d$ .
2. Write down  $dn + c$ .
3. Plug in any value of  $n$  for which you were given a value, and  $c$  is what makes the value in the sequence come out right.

Example

2, 11, 20, 29, ...

$d = 9$ , so we write

$$9n + c$$

Now let's choose  $n = 1$ . The value should be 2, so  $c$  needs to be -7 to make it right. Therefore the formula for this arithmetic sequence is

$$9n - 7$$

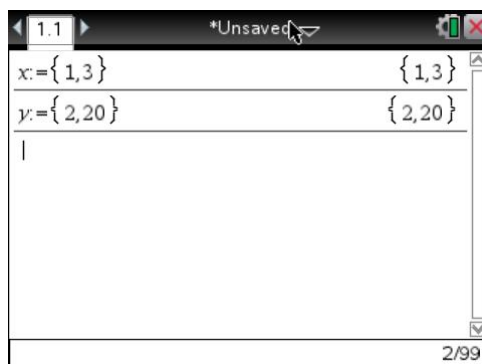
This is how you could do this on the TI-Nspire.

We think of  $n$  as being our  $x$  variable and the sequence value as being our  $y$  variable. So, in the above example we would have the points

(1,2), (2,11), (3,20), (4,29)

We only need two of them, and any two will do. For clarity I'm going to choose the first and the third.

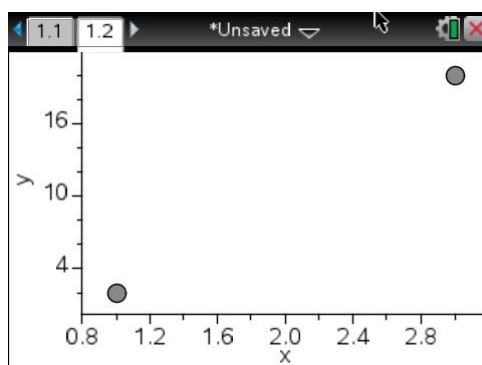
Open up a calculator app and make a list (which I've called  $x$ ) of the first and third  $x$  values. Now make another list (which I've called  $y$ ) of the first and third  $y$  values. Make sure you choose the numbers in the same order in each list. Here's the screen.



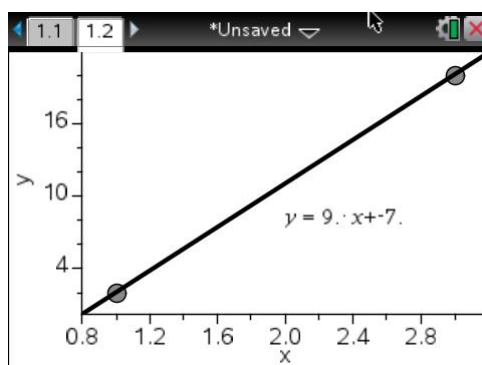
Now add a Data & Statistics app. We get the following screen



Do as the screen says. Hover the pointer over the middle of the bottom, click, and choose the variable  $x$ . Now hover the pointer over the middle of the left, click, and choose  $y$ . This is what you get.



We're almost done. Choose **[menu][Analyse][Regression][Show Linear ( $mx + b$ )]** to get



The bit we want is the  $9.0x - 7.0$ .

In case you haven't guessed, this is the Nspire's way of writing  $9.0 \times x - 7.0$

Because screen space is limited, the Nspire has left off the trailing zeros, and as you may have noticed before, the Nspire represents multiplication with a “.” So what this says is  $9x - 7$ , but for an arithmetic sequence we would replace the  $x$  with a  $n$ , as shown earlier in this section.

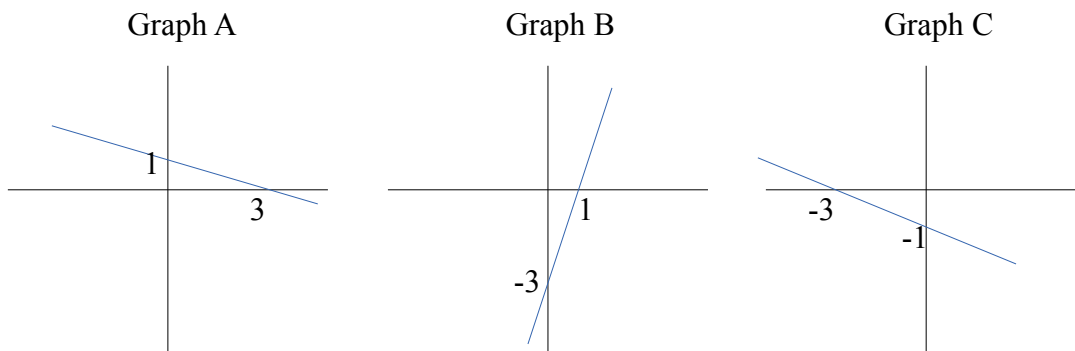
## 5. Graphs

### 5.1 Straight Line Graphs

There are a variety of problems involving straight line graphs. Some common ones that the TI-Nspire can help you with are:

- Matching the picture of a graph to its equation
- Finding the  $x$  and  $y$  intercepts of a line
- Finding the equation of a line through two points
- Finding the equation of a line through one point, when you know its gradient

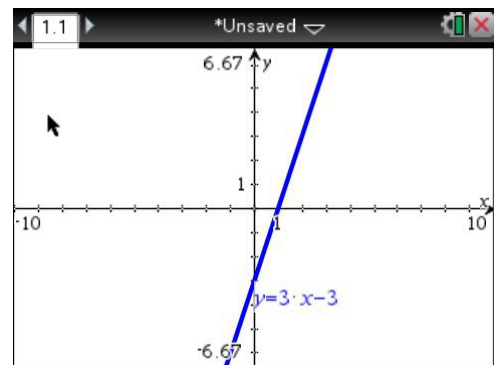
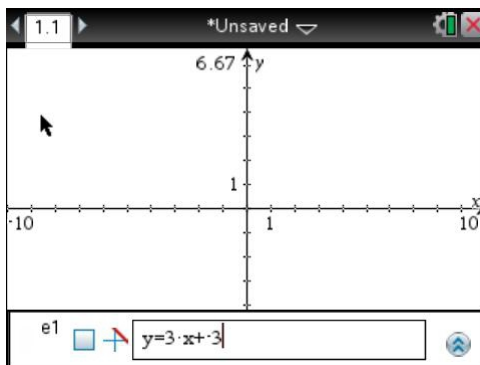
A typical matching problem would be the following.

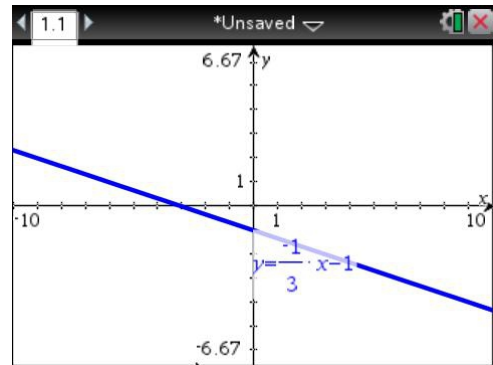
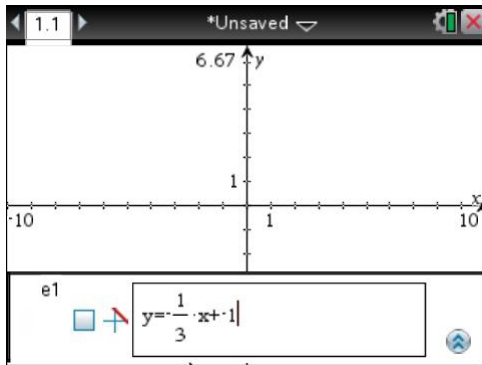
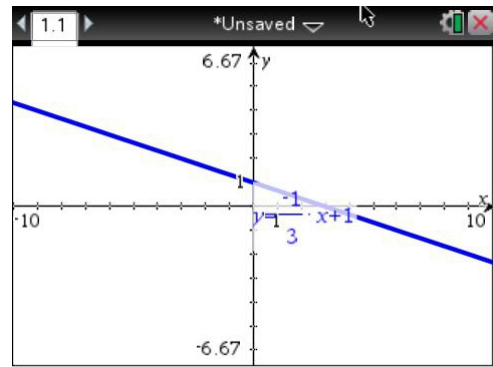
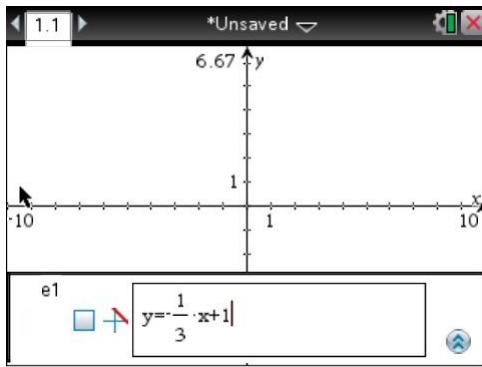


Equation 1  $y = 3x - 3$   
Equation 2  $y = -\frac{1}{3}x + 1$   
Equation 3  $y = -\frac{1}{3}x - 1$

Match each graph to its equation.

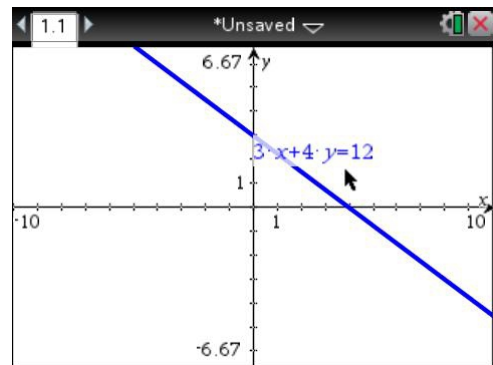
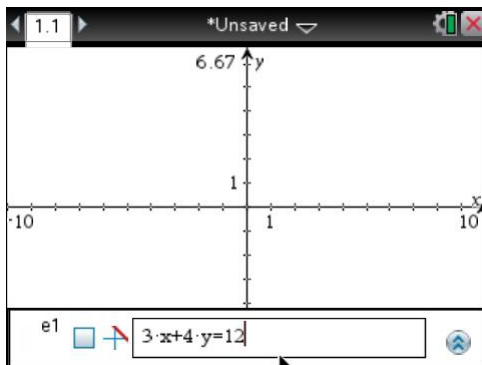
To plot lines given in this form we use [menu][Graph Entry/Edit][Equations][Line][ $y=m \cdot x+b$ ]. here are the entry screens and graphs for each equation.





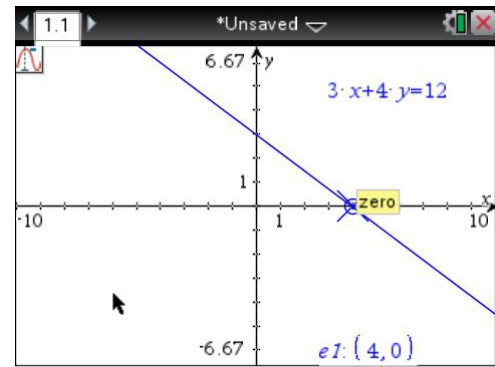
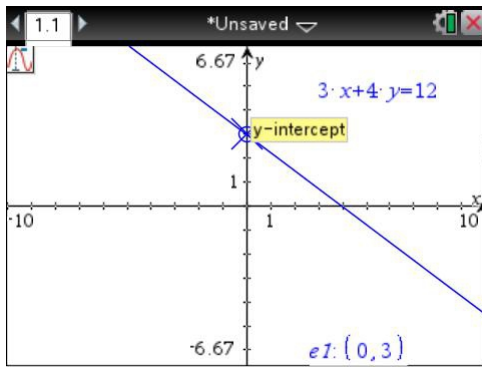
It is now obvious that the correct matches are A2, B1 and C3.

Now lets see how to find the  $x$  and  $y$  intercepts of a line. We will do this for the line  $3x + 4y = 12$ . For a line written in this way we use **[menu][Graph Entry/Edit][Equations][Line][ $a \cdot x + b \cdot y = c$ ]**. Here are the entry screen and the graph.



To find the intercepts we choose **[menu][Trace][Graph Trace]**. A circled  $\times$  will appear somewhere on the line. We can move this along the line by using the left and right arrows on the calculator's touch pad. If you look at the following two screens you will see that the Nspire tells us when we are on an intercept. When we are on the  $y$  intercept it says exactly this. When we are on the  $x$  intercept it displays the word "zero". In both instances it also displays the coordinates of the intercept.

We can see, from the following two screens, that the  $y$  intercept is  $(0,3)$  and the  $x$  intercept is  $(4,0)$ .



For our next two problems we won't be drawing graphs. We will be using a couple of functions we have already encountered in the Algebra section of this book.

Imagine that we need to find the equation of a line that goes through the points  $(-1, 2)$  and  $(3, 10)$ . We know that both these points must satisfy an equation of the form  $y = mx + c$ . Substituting into this equation gives us the simultaneous equations

$$2 = -m + c \quad \text{and} \quad 10 = 3m + c$$

We can use `linSolve` to find  $m$  and  $c$ , as shown below.

```
linSolve({2=-m+c,10=3*m+c},{m,c})
{2,4}
```

So, the equation of the line through the points  $(-1, 2)$  and  $(3, 10)$  is  $y = 2x + 4$ .

For our last problem in this section we are going to find the equation of a line, of gradient  $-3$ , through the point  $(1, 2)$ . This point must satisfy the equation  $y = -3x + c$ . And substituting in gives us  $2 = -3 + c$ . Of course, many students will be able to do this in their head, but if the numbers were a bit awkward we could use `nSolve`, which shows us that the equation we want is  $y = -3x + 5$ .

```
nSolve(2=-3*1+c,c)
5.
```

## 5.2 Non Linear Simultaneous Equations

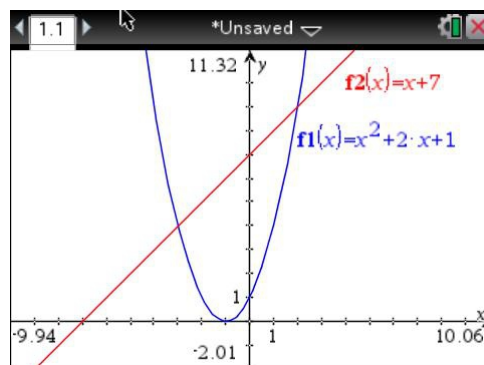
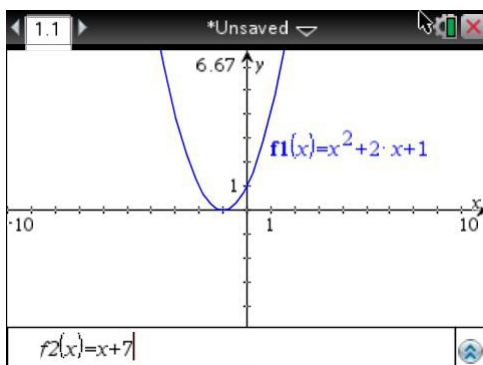
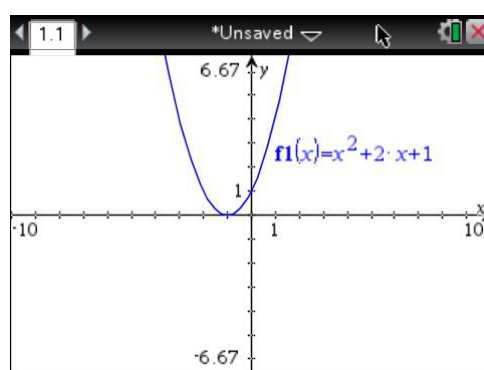
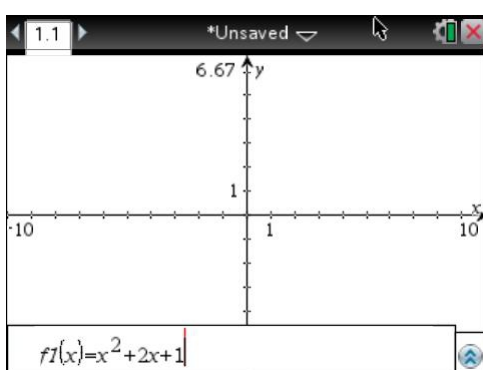
These are among the harder problems you may encounter. The standard way in which you are expected to solve them is by first making a substitution, and then manipulate this to give you a quadratic equation in one of the variables. However, the graph app on your TI-Nspire allows you to take another approach.

Here is one of the simplest of such problems that you might come across.

Find the solution(s) to the simultaneous equations

$$y = x^2 + 2x + 1 \quad \text{and} \quad y = x + 7$$

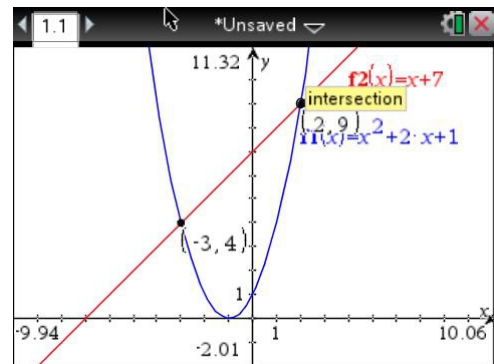
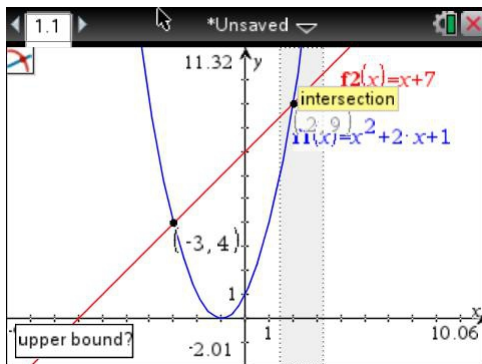
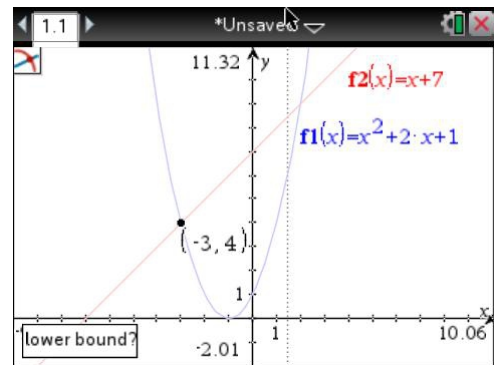
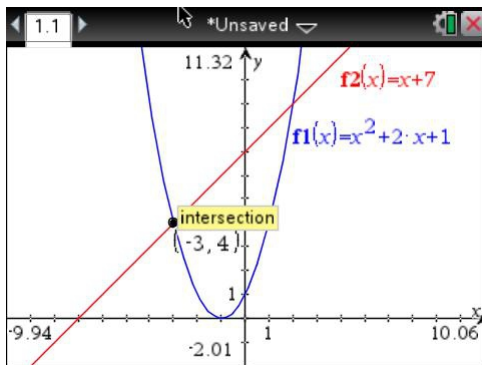
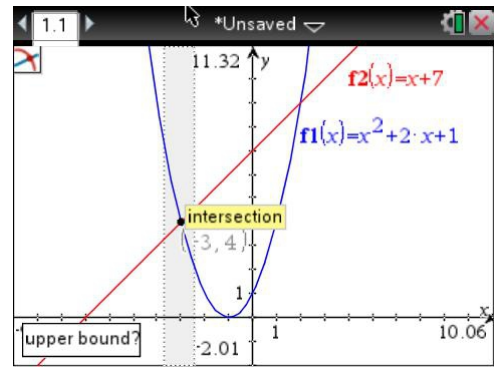
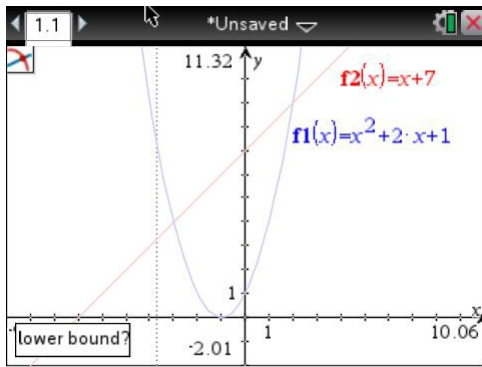
We plot the two graphs on the same screen, as shown below. Don't forget that the quickest way to display the " $f(x)=$ " entry line is **[Ctrl][G]** – exam time is precious.



Notice, I've moved the axes down, so that I can see both intersections. Alternatively, I could have zoomed out.

To find the coordinates at the intersection points we use **[menu][Analyse Graph][Intersection]**. You need to do this for each intersection point. You will be asked for a lower bound, which should be immediately to the left of the point you are interested in. Use the touch pad left and right arrows, or simply swipe your finger left or right on the touch pad; when you are happy press **[enter]**. Now you are asked for an upper bound, which should be immediately to the right of the point you are interested in. After you have pressed enter a dot will appear at the intersection point, and its coordinates will be displayed. The following six screens show the procedure for our two intersection points.





You can now see that the solutions to our equations are:  $x = -3, y = 4$  and  $x = 2, y = 9$ .

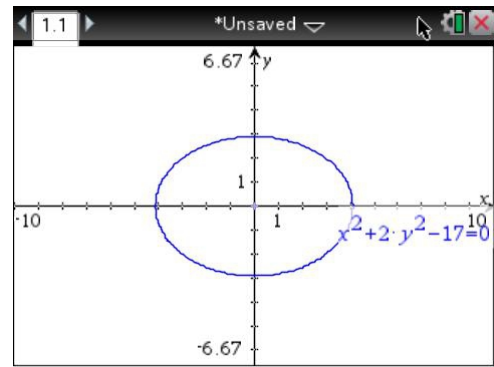
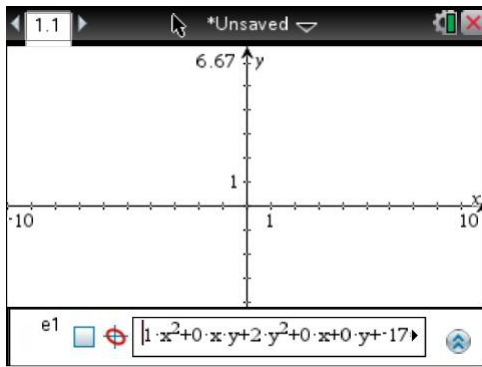
You may, however, be given a less straightforward problem, such as the following one.

Find the solution(s) for the following pair of equations

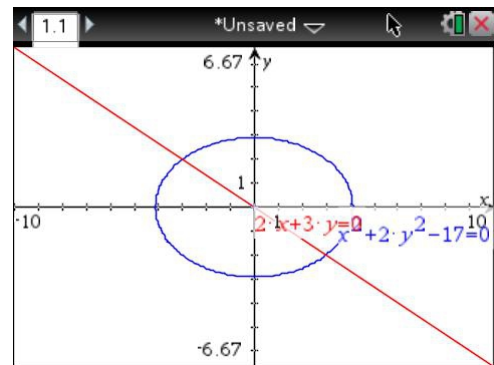
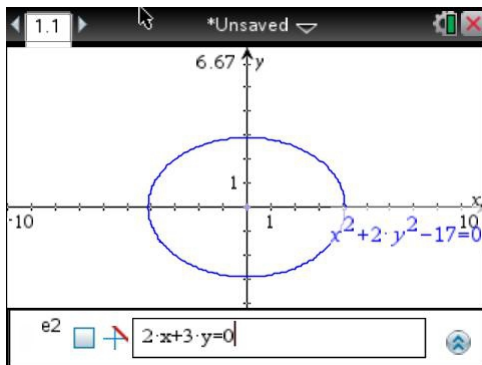
$$x^2 + 2y^2 = 17 \quad \text{and} \quad 2x + 3y = 0$$

To plot a graph of the first equation we rewrite it as  $x^2 + 2y^2 - 17 = 0$ .

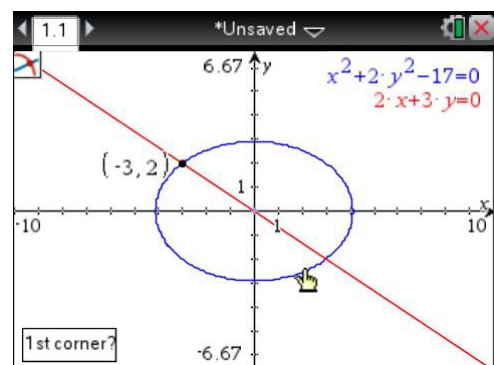
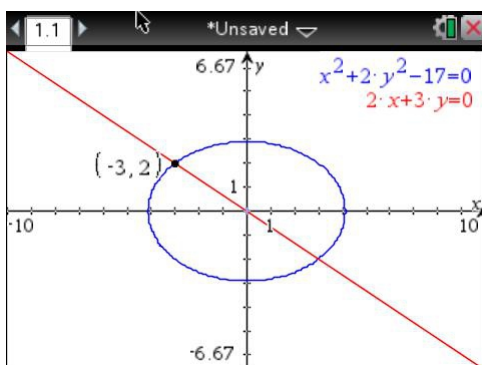
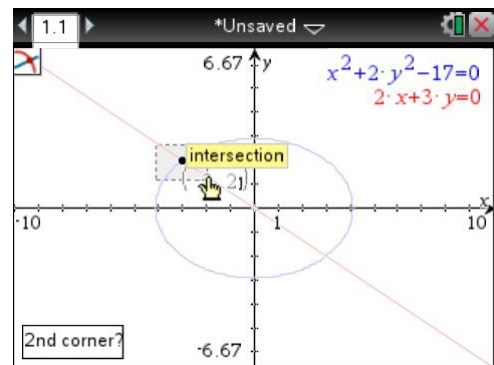
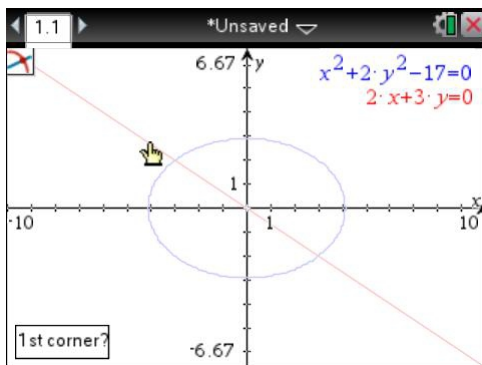
Then we use [menu][Graph Entry/Edit][Equation][Conic]. Here are the two screens showing this.

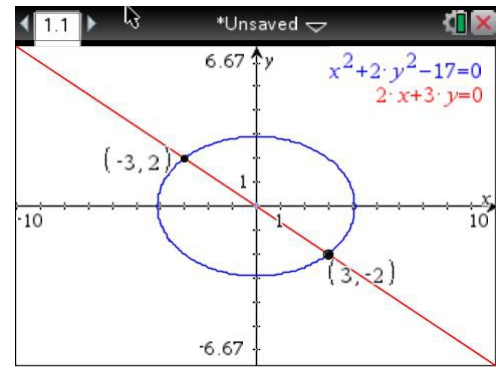
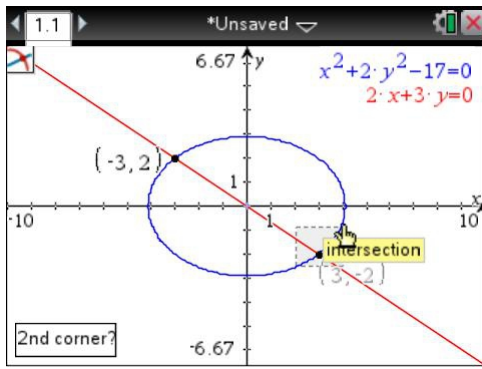


Now, for the second equation we use [menu][Graph Entry/Edit][Equation][Line][ $a \cdot x + b \cdot y = c$ ], as shown below.



We can now find the intersections, in the same manner as the last example, but this time we are asked for a 1<sup>st</sup> corner and a 2<sup>nd</sup> corner. One corner should be to the upper left of the intersection, and the other to its lower right: it doesn't matter which way round. Alternatively, one corner should be to the lower left, and the other to the upper right. Move the hand to indicate these positions. The whole procedure is shown below.

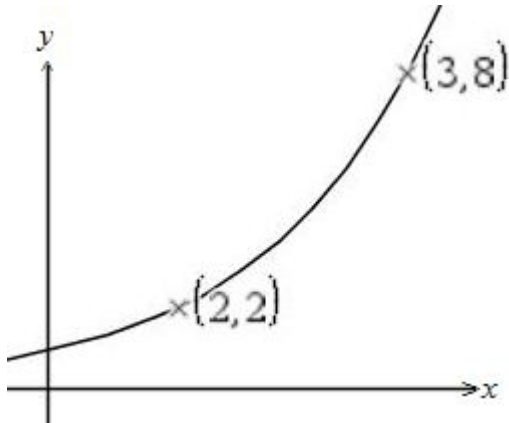




As you can see, the solutions to our equations are  $x = -3, y = 2$  and  $x = 3, y = -2$ .

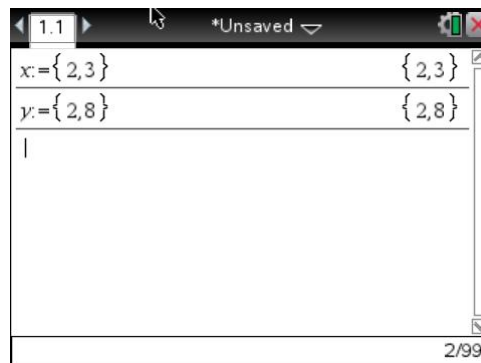
### 5.3 Exponential Graphs

While these don't show up on every exam paper, when they do your TI-Nspire can sort them out easily. You might get a question like this.

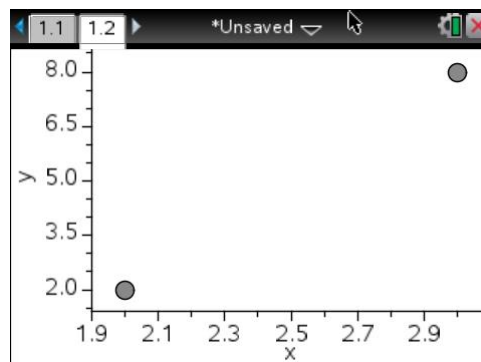


This is a sketch of the graph  $y = ka^x$ , which passes through the points (2,2) and (3,8). Find the coordinates of the point P, on the curve, where  $x = 2.5$ .

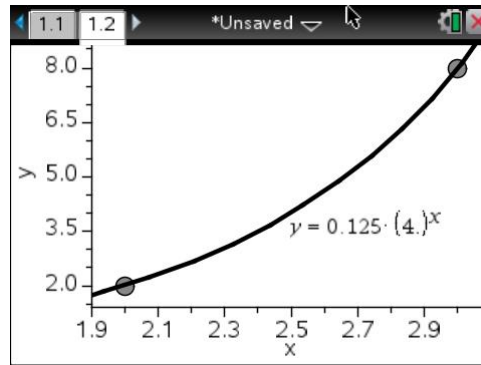
We are going to use exponential regression to find the values of  $k$  and  $a$ . First we start with the calculator app. We create a list of  $x$  values, and a corresponding list of  $y$  values, keeping the values in the same order, as in the screen shot below.



Now we add a Data and Statistics app, and assign the  $x$  and  $y$  lists to their appropriate axes, as has already been described in the earlier sections of this book.



Finally, we choose **[menu][Analyse][Regression][Show Exponential]**. This is what we get



So,  $k = 0.125$  and  $a = 4$ . Therefore the equation for the curve is

$$y = 0.125 \times 4^x$$

or

$$y = \frac{4^x}{8}$$

if you prefer.

Going back to the calculator, and substituting 2.5 into the formula we get

$$\frac{4^{2.5}}{8} = 4$$

So the coordinates of P are (2.5,4)

## 5.4 Inequalities

Perhaps you have noticed that if you press **[del]**, when you first get the entry line for a graph, as well as the equals sign disappearing you are forced to choose one of  $\leq$ ,  $<$ ,  $=$ ,  $>$  or  $\geq$ . Choosing anything other than  $=$  causes the  $f(x)$  to be replaced by  $y$ : if you wish you can use the left arrow to skip over whichever inequality symbol you have chosen, and replace the  $y$  with an  $x$ . Anyway, having chosen an inequality symbol, our Nspire can now plot inequalities. The region satisfying the inequality is shaded. If the line (or curve) delineating the region should be included (for  $\leq$  and  $\geq$ ) then it is solid. If the line is not part of the region (for  $<$  and  $>$ ) it is dashed. You almost certainly already use this convention.

We can plot several inequalities at once. The region which satisfies all the inequalities will have overlapping shading, and so it will be the darkest. However, you should look fairly carefully to make sure you really have chosen the region that has the most overlapping.

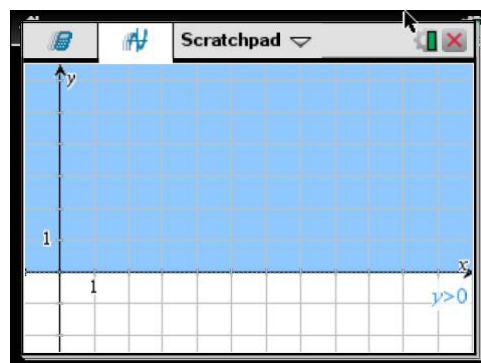
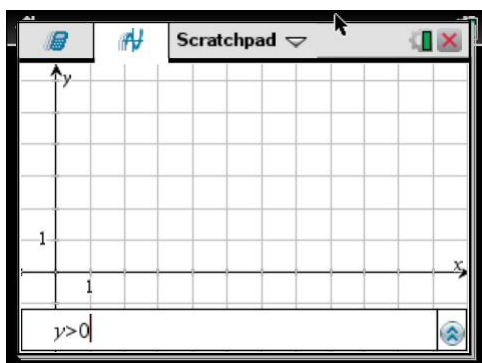
Here's a typical problem

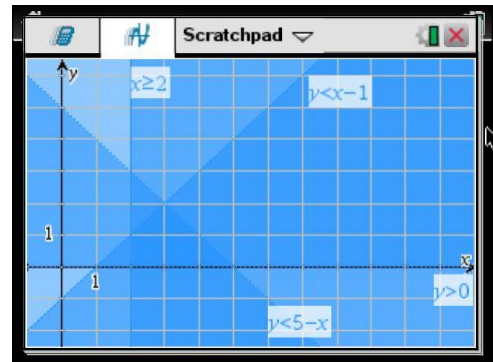
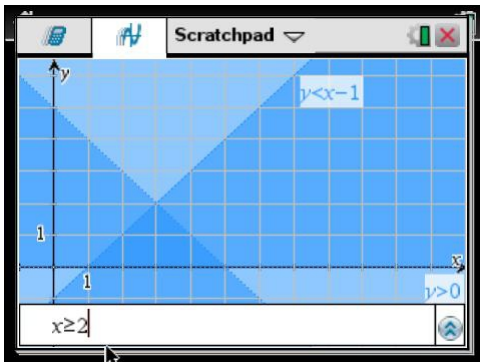
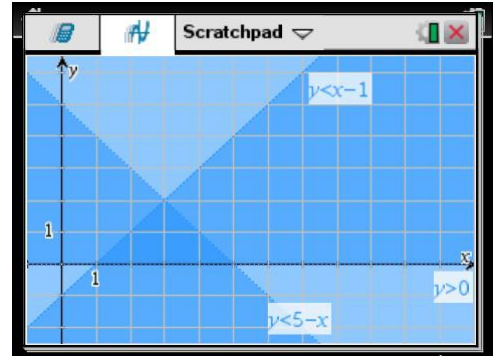
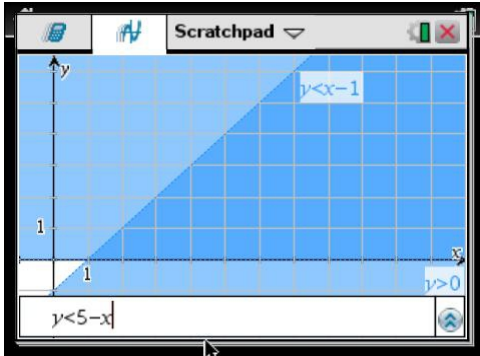
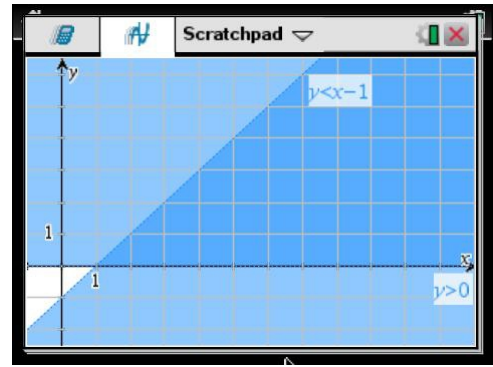
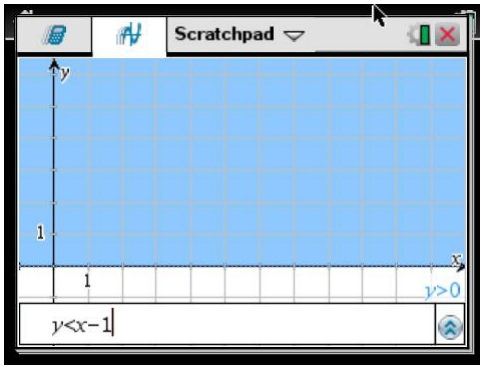
On the grid below, shade in the region defined by the inequalities

$$y > 0, \quad y < x - 1, \quad y < 5 - x, \quad x \geq 2$$

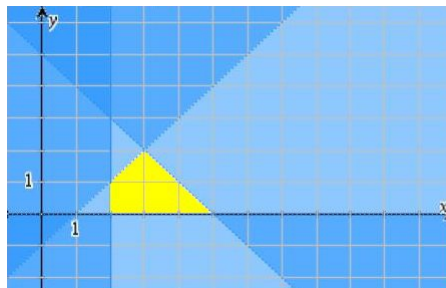


The following eight screens show each of these inequalities being plotted.





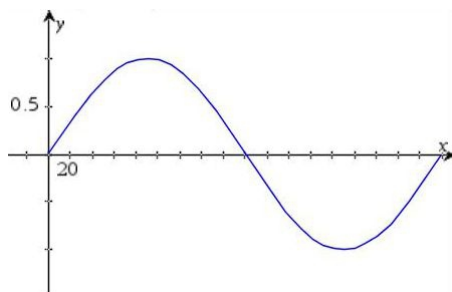
As you can (or maybe can't) see, picking out the darkest region is starting to get a bit hard. So I've shown the region that we need to colour in for the examiner below: it's in yellow.



## 5.5 Transforming Graphs

Questions on this topic, when they occur, are aimed squarely at candidates who might expect to get an A. Well, I'm going to show you how to use the TI-Nspire so that picking up these marks becomes a cinch. The best way to do this is with a couple of examples.

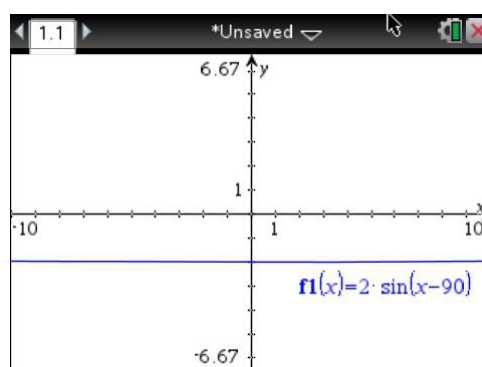
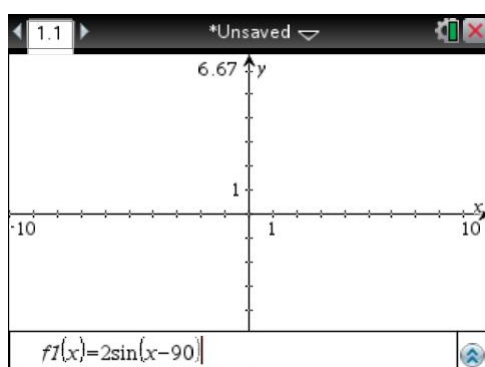
Let's get the most straightforward one out of the way first.



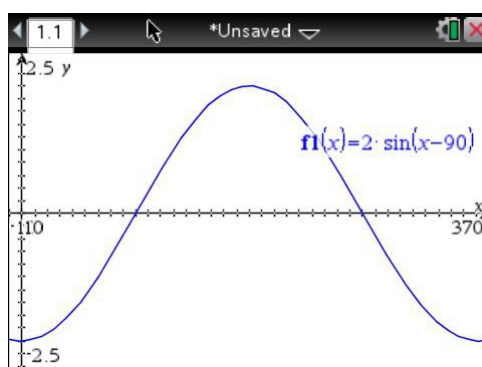
On the left is a graph of  $y = \sin x^\circ$ , for values of  $x$  from  $0^\circ$  to  $360^\circ$ .

Sketch the graph of  $y = 2\sin(x - 90)$ , for values of  $x$  from  $0^\circ$  to  $360^\circ$ .

The first thing you should do, whenever you are using trigonometric functions, is make sure that your calculator is in the right mode: that is, set to degrees, in this case. Move the pointer just to the left of the battery status symbol and the Nspire will tell you if it's working in degree or radians mode – **you never want to be using gradians**. If not, use **[menu][Settings]** and set the Graphing Angle to degrees. Let's now plot the graph we were asked for.



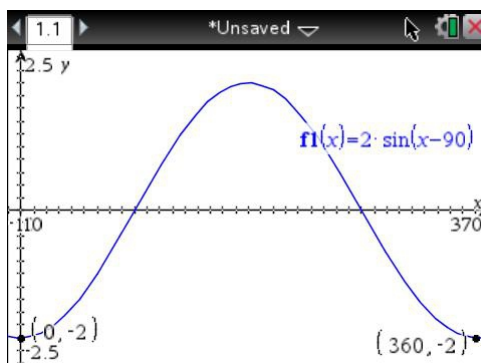
On the default scale it doesn't look too informative, so we will use **[menu][Window/Zoom][Window Settings]** to improve things. We can leave Xmin at -10, but change XMax to 370. Also, change YMin to -2.5, and YMax to 2.5.



That's a lot better!

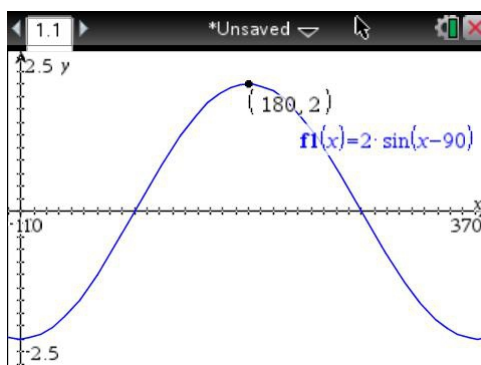


Now our examiner wants to see any special features of the graph, and they had better be in the right place. Using **[menu][Trace][Graph Trace]**, as described earlier, will pick out the  $y$  intercept and any  $x$  intercepts. In our example the  $y$  intercept is  $(0, -2)$ , and we have  $x$  intercepts at  $(90, 0)$  and  $(270, 0)$ . We also need to check out any minimums, by using **[menu][Analyse Graph][Minimum]**. How to do this was described earlier. Here's what we get.



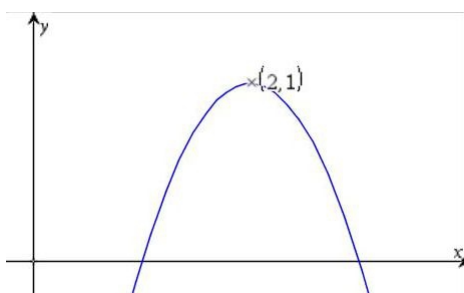
As you can see, one is at  $(0, -2)$ , which is the same as the  $y$  intercept, and another is at  $(360, -2)$ .

In a similar fashion, using **[menu][Analyse Graph][Maximum]**, we get any maximum points.



As you can see, there is just one at  $(180, 2)$ .

We now come to what I think is one of the coolest tricks in this book. Say I'm given a function  $f(x)$  and told that it has a minimum at coordinates  $(a, b)$ . I can produce one such function by entering  $f(x) := (x - a)^2 + b$ . If instead I needed a maximum I'd use  $f(x) := -(x - a)^2 + b$ . I don't care what the rest of the graph looks like (in fact, it's a parabola) because I (and the examiner) am only interested in how this minimum, or maximum, transforms. Let's see how this works with an example.

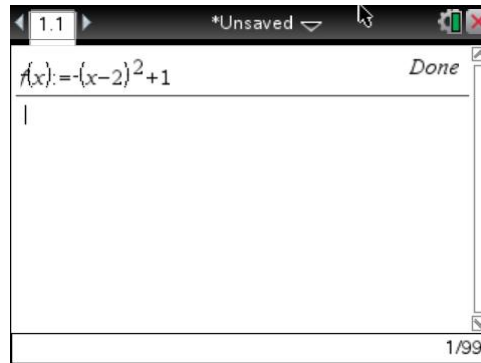


The curve  $y = f(x)$  has a maximum point at  $(2, 1)$ .

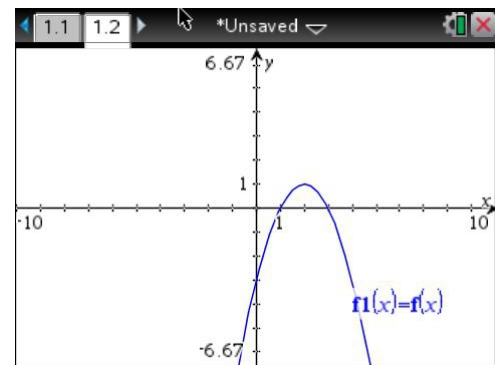
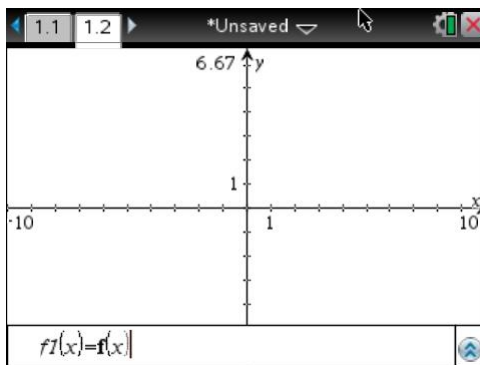
Write down the coordinates of the maximum point for the curve:

- a)  $y = f(x+3)$
- b)  $y = 2f(x)$
- c)  $y = f(2x)$

First we use the calculator app to define a suitable function  $f(x) := -(x - 2)^2 + 1$ .

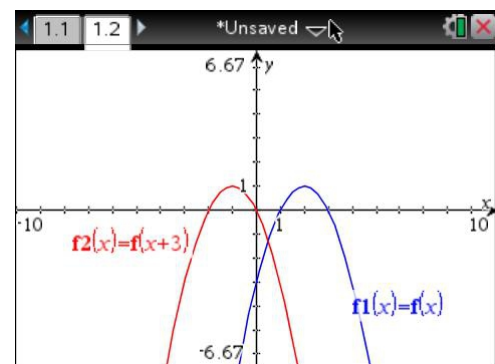
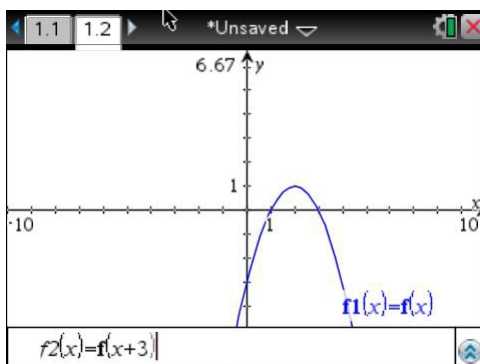


Now let's plot it.



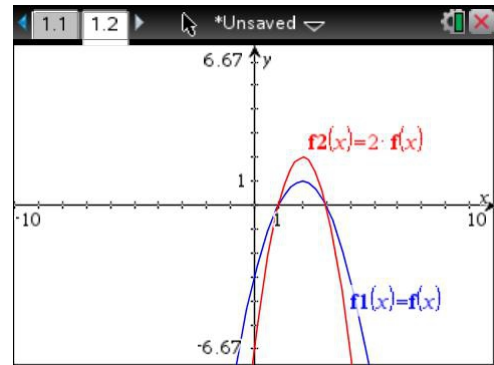
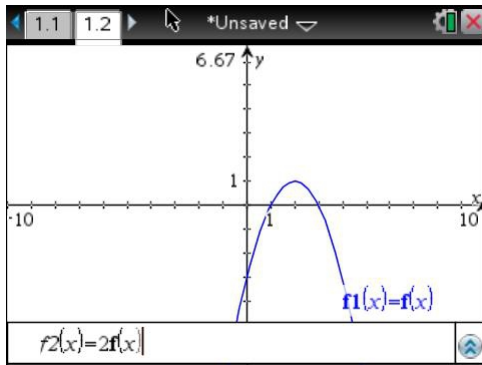
Yes, we've got a maximum in the correct place.

For a we plot  $f(x+3)$



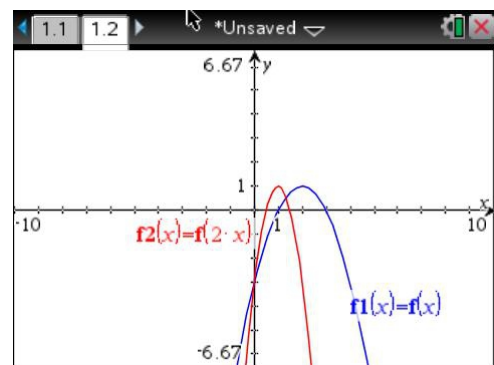
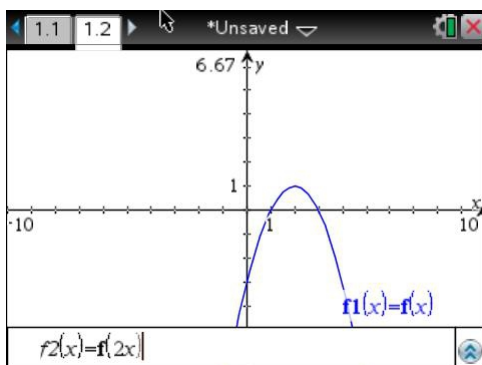
We can see that the maximum point is at (-1,1).

For b we plot  $2f(x)$



The maximum is at (2,2).

For c we plot  $f(2x)$



The maximum is at (1,1).

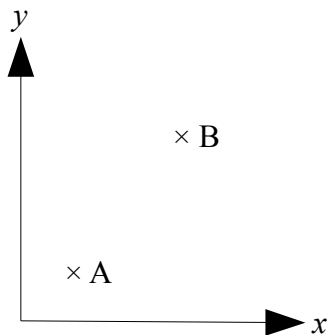
Even strong candidates sometimes get a bit confused by this topic, but if you can remember how to set up  $f(x)$  as I've shown then getting these marks (intended for top candidates) is like taking candy from a baby.

## 6. Vectors

I considered leaving this short chapter out, as relatively few questions come up at GCSE and IGCSE where this stuff can be used. However, they do occasionally crop up, and they may also crop up more frequently in other exams that students reading this book might be taking. So, it's here for those who are interested.

A lot of vector questions are purely algebraic, in which case any calculator you are allowed bring into your exam isn't going to help you. But, what follows are a few examples of the type of GCSE questions that have been set, and that your TI-Nspire might help you with.

The following problem is very similar to one that occurred on a fairly recent paper.



In the sketch to the left A is the point (3,1) and B is the point (5,5).

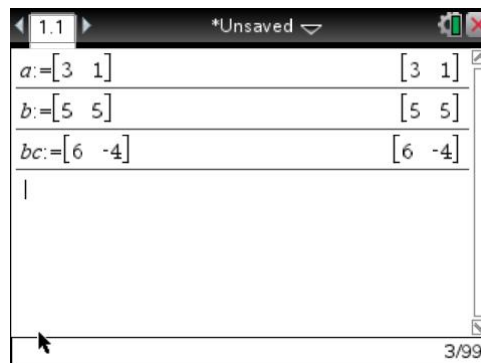
- a) Find the vector  $\overline{AB}$ , giving your answer in column form.

$$\overline{BC} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}, \quad \text{D is the midpoint of AB,}$$

E is the midpoint of BC.

- b) Find the vector  $\overline{DE}$ , giving your answer in column form.

Let's first define our vectors. The Nspire uses square brackets to do this. In the screen below I've defined *position vectors*  $\mathbf{a}$  and  $\mathbf{b}$ , and the *displacement vector*  $\mathbf{bc}$ .



For the first part of the question, we want the displacement vector  $\overline{AB}$ , which is simply  $\mathbf{b} - \mathbf{a}$ . You can see the resulting screen on the next page.

1.1 *Unsaved	
$a:=[3 \ 1]$	$[3 \ 1]$
$b:=[5 \ 5]$	$[5 \ 5]$
$bc:=[6 \ -4]$	$[6 \ -4]$
$b-a$	$[2 \ 4]$
4/99	

We were asked to write this in column form, so our answer is

$$\overline{AB} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

Position vectors D and E, which I've defined as  $d$  and  $e$ , are calculated as show in the next screen.

1.1 *Unsaved	
$bc:=[6 \ -4]$	$[6 \ -4]$
$b-a$	$[2 \ 4]$
$d:=\frac{1}{2} \cdot a + \frac{1}{2} \cdot b$	$[4 \ 3]$
$e:=b + \frac{1}{2} \cdot bc$	$[8 \ 3]$
6/99	

The second part of the question is now worked out in the same manner as the first part.

1.1 *Unsaved	
$b-a$	$[2 \ 4]$
$d:=\frac{1}{2} \cdot a + \frac{1}{2} \cdot b$	$[4 \ 3]$
$e:=b + \frac{1}{2} \cdot bc$	$[8 \ 3]$
$e-d$	$[4 \ 0]$
7/99	

Once again, we are asked for our answer in column form.

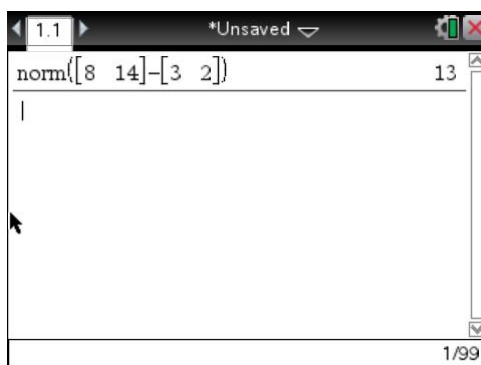
$$\overline{DE} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

We can also use vectors to work out the distance between two points, in both 2 dimensions and 3 dimensions. To do this we use the norm function.

Here's a 2D example.

Find the distance between the points (3,2) and (8,14)

The answer is given by the norm of the displacement vector between these points.



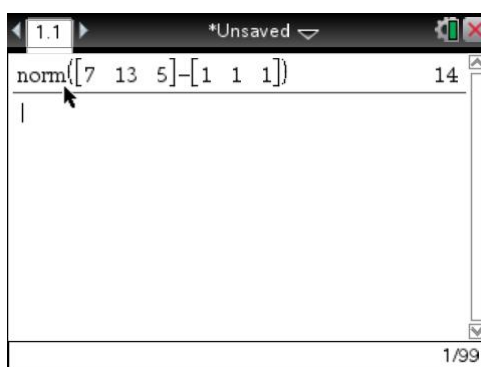
Actually, I entered  $\text{norm}([8,14]-[3,2])$ . The calculator has redisplayed this with spaces replacing the commas. The Nspire separates vector components with spaces, but when you enter a vector you **must** use commas to separate each component.

Finally, here is a 3D example.

One corner of a cuboid has coordinates (1,1,1). Its diagonally opposite corner has coordinates (7,13,5).

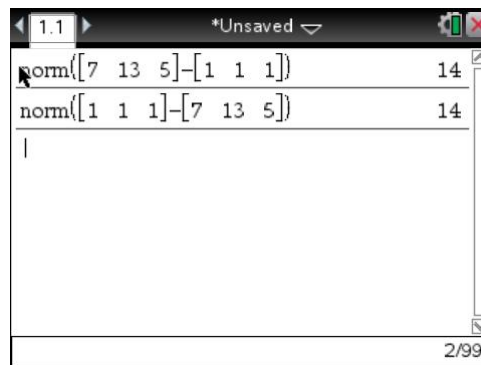
Find the length of this diagonal.

This time we need the norm of a 3D displacement vector. Here's what it looks like on the calculator.



Once again, what I entered was  $\text{norm}([7,13,5]-[1,1,1])$ .

In case you are worrying about it, for the last two questions, the order of the position vectors doesn't matter. Here is the 3D one again, with the coordinates the other way round.



Don't forget that you might have to show your working to get full marks. So for the 3D example you would need something like

$$\sqrt{(7-1)^2+(13-1)^2+(5-1)^2}=14$$

There's certainly some scope for a typo, or another slip up, so what I've shown you allows you to quickly check that you have got the correct answer.

*And that, folks, brings us to the end of this book.*