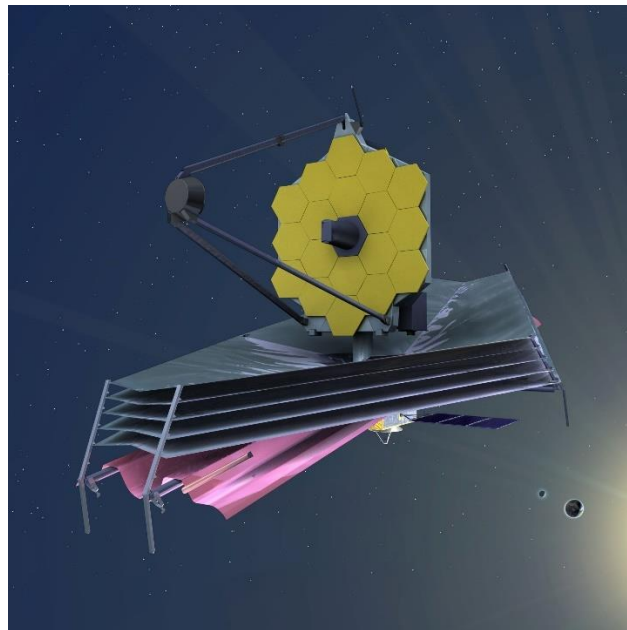


Calculating the position of the James Webb Space Telescope using Python or TI-Basic

Teacher and Student Notes

1. Science Background



Source: NASA

The James Webb Space Telescope (JWST) has been designed to look far back in time to the earliest galaxies formed after the dark ages period in the formation of the Universe. This was about a billion years after the big bang which itself was about 13.8 billion years ago. Since then the Universe has been expanding and the light from these early galaxies has stretched into the infrared part of the spectrum which is why the JWST is observing in the infrared. Any stray heat (thermal energy) is likely to swamp the sensitive detectors, hence the layers of shielding and the positioning at Lagrange point 2. Here at Lagrange2 the telescope will have the same orbital period as the Earth but be 1.5 million kilometres away on the far side of the Earth from the Sun, always maintaining a position keeping the telescope protected from the glare of the Sun. Communications are also easy to maintain between Earth and the JWST as it maintains a constant position relative to the Earth. The mirrors themselves are covered in a thin layer of gold to optimise them for reflecting infrared light. (Gold nano-particles are used in some cancer treatments as they are good at absorbing infra-red and can be located close to the tumour itself).

The Lagrange points (and there are five of which three lie on a straight line through Sun and Earth) are gravitational anomalies. Normally anything orbiting closer to the Sun will orbit faster and anything further away will move more slowly. Mercury for example orbits in 80 days and Pluto takes 250 years for one complete orbit. But the Earth's gravitational field detracts from that of the Sun on the Sun side and adds to that of the Sun on the far side. This means that instead of the gravitational field increasing all the way to the Sun, there is a point where the field is reduced just enough for an orbit of one year, to match the Earth. Likewise moving away from the Sun, the Earth's field adds to that of the Sun and there is a point where the field again will allow a solar orbiting object to orbit with a period of one year, this is Lagrange2.

2. Connection to the Sustainability Goals.

Construction and operation of the James Webb Space Telescope will have a major impact on the design and operation of further space-based platforms for observing the Earth. Spinoff from the infrared cameras and cooling devices will all play a part in health and the environment here on Earth. There is a big demand for reliable data to inform us about global warming. Land use has benefitted from the series of Landsat satellites over the years and it should not be forgotten that without satellite observations we would not have discovered the hole in the ozone layer. Meteorologists rely on satellites for detailed data on wind cloud and temperature for weather forecasting. Global positioning systems are used for fishing and agriculture in addition to their more widely known use for travelling. Without communication networks we would not have the communication systems we enjoy today which help in the search for a better and safer environment for us all.

SDG 13, 14 15 for climate, oceans and land use.

SDG 4 and 6 for education and clean water.

SDG 3 for health and well being

3. Student Learning Outcomes

- Learn about the development of a large scientific project (JWST)
- Able to identify and understand what the Lagrange points are
- Able to use their knowledge of gravitation to calculate the position of a Lagrange point
- Able to use their knowledge of programming to solve difficult equations.

4. The Task: Teacher Hints

a. TI Basic:

If using the tns file Lagrange3 then the YouTube video “*Solving equations with TI-Nspire (nSolve)*” at https://www.youtube.com/watch?v=z_0MHWyIN7Y might be helpful for understanding how nsolve() works in TI-Basic. It is essentially an iterative technique for finding a solution to an equation with a single unknown variable.

Having set up the function for the centripetal acceleration, a single statement in the calculator page

```
rs:=nSolve(acc(r)=0,r,1000000)
```

is sufficient to solve the function for r.

b. Python:

If using the tns file Lagrange2 then it is helpful to know that the programming technique used here is known as a recursive divide and conquer algorithm.

Having defined the function for the centripetal force $acc(r)$, the zero function for carrying out the recursive search is then defined, $zero(lt,rt)$.

First the meaning of the variables:

- lt – left....a lower bound for the variable r
- rt – right...an upper bound for the variable r
- m – center....or midpoint between lt and rt.

Recursive description:

A guess is made for the lower and upper bounds of r so that r_s (r_s) lies between these boundaries

```
rs = zero(rges + 1e+06, 2 * rges)
```

Find the zero in the interval [lt, rt] by first determining the interval center m. If the difference between the interval boundaries is less than epsilon (eps), return m as an approximation of the zero. Otherwise, check whether the function values of the $acc(r)$ function have opposite signs at the interval boundaries. If this is true, find the zero in the left subinterval [lt; m], otherwise find the zero in the right subinterval [m; rt].

5. Suggestions

The pdf file titled “The Lagrange points in the system Sun-Earth” could be given to students so that they can identify the relevant variables. The teacher should talk through the way in which the Lagrange points arise and if necessary explain how to set up the equation for the centripetal force at Lagrange1. The notes at https://resources.t3europe.eu/t3europe-home?resource_id=3487 might be helpful.

Show how the velocity of the satellite (v_{tel}) is eliminated by equating the period of the satellite to the period of the Earth.

r_s is set to r to set up the function $f(r)$. Students should be asked to consider how the equation $f(r)=0$ might be solved!

Teachers could ask students to set up the equations for the centripetal acceleration at each of the Lagrange points.

If using TI-Basic, students could be asked to set up a recursive technique similar to the technique used in the Python program and compare any result to that obtained using *nsolve*.

Students could be asked to construct an algorithm or flow chart explaining how the function $zero(lt,rt)$ works.

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