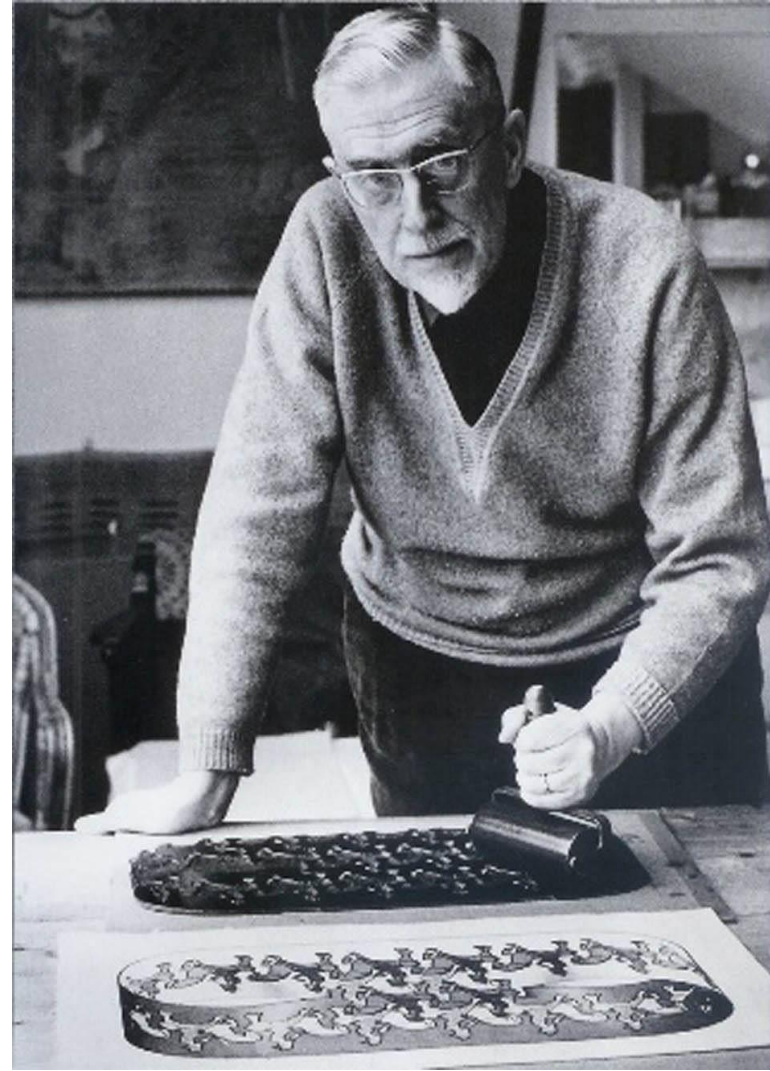


Maurits Cornelis Escher (1898-1972)

Leeuwarden, June 17



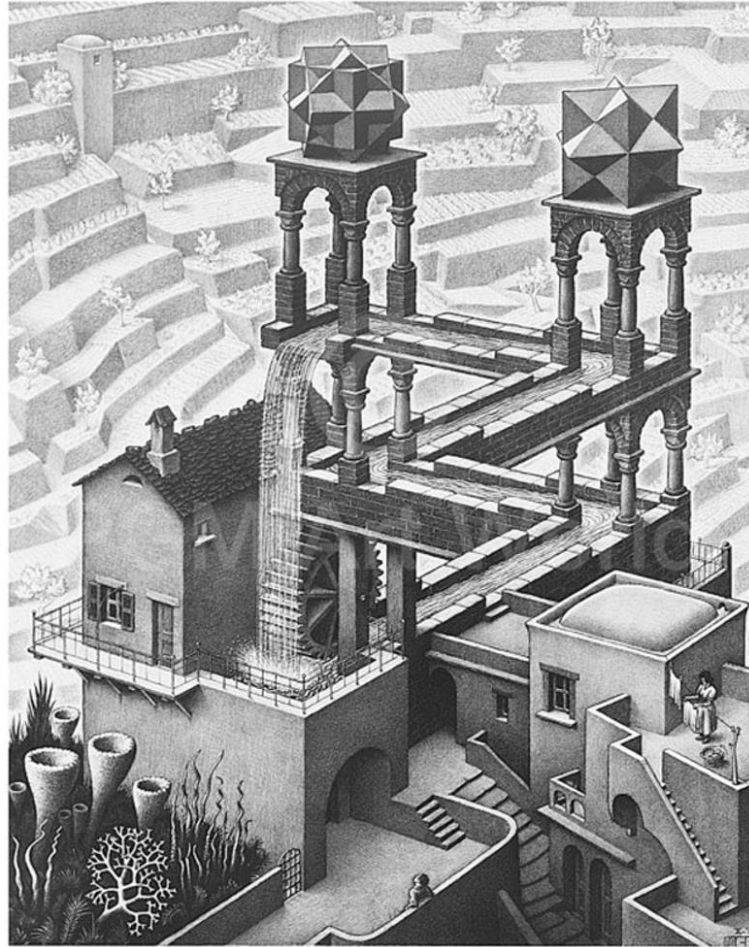
Escher

Graphical artist

Escher

Graphical artist

Optical illusions



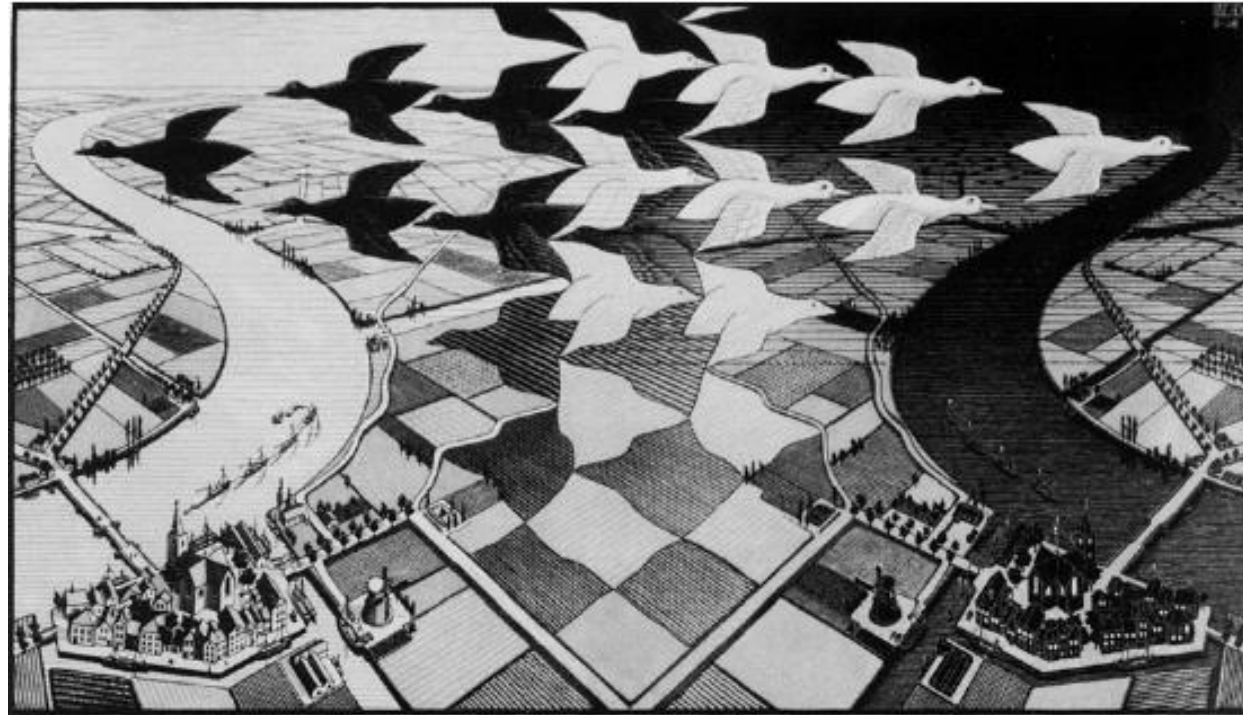
"Waterfall", 1961 Lithograph

Escher

Graphical artist

Optical illusions

Transformations



"Day and night", 1938 woodcut

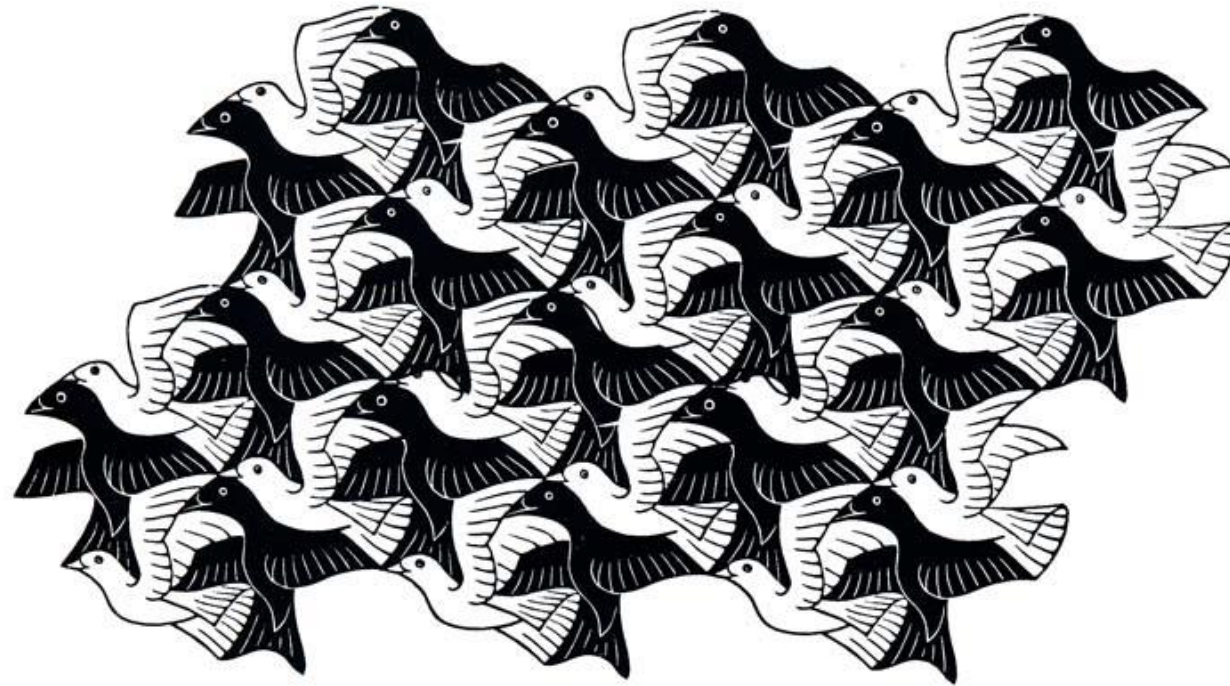
Escher

Graphical artist

Optical illusions

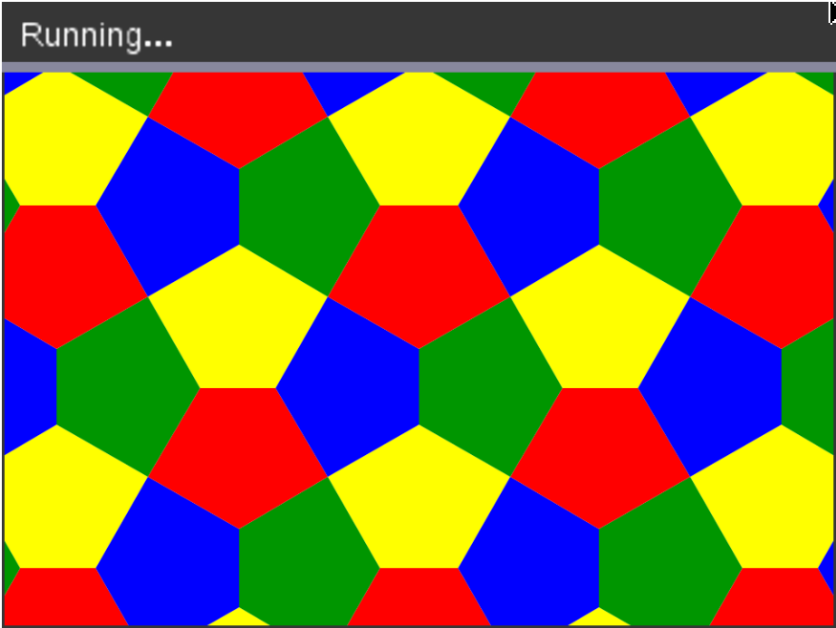
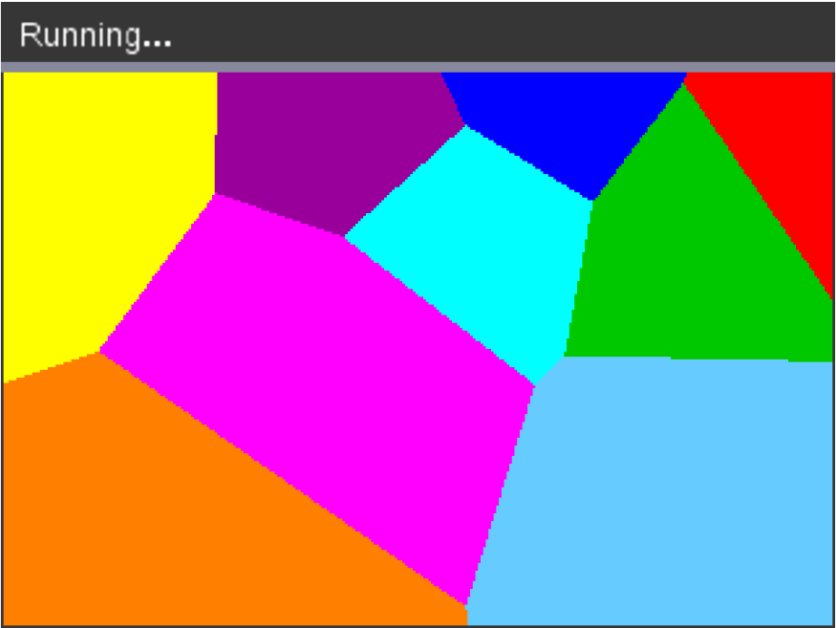
Transformations

Tessellations



"Birds", 1949 woodcut

Tessellations

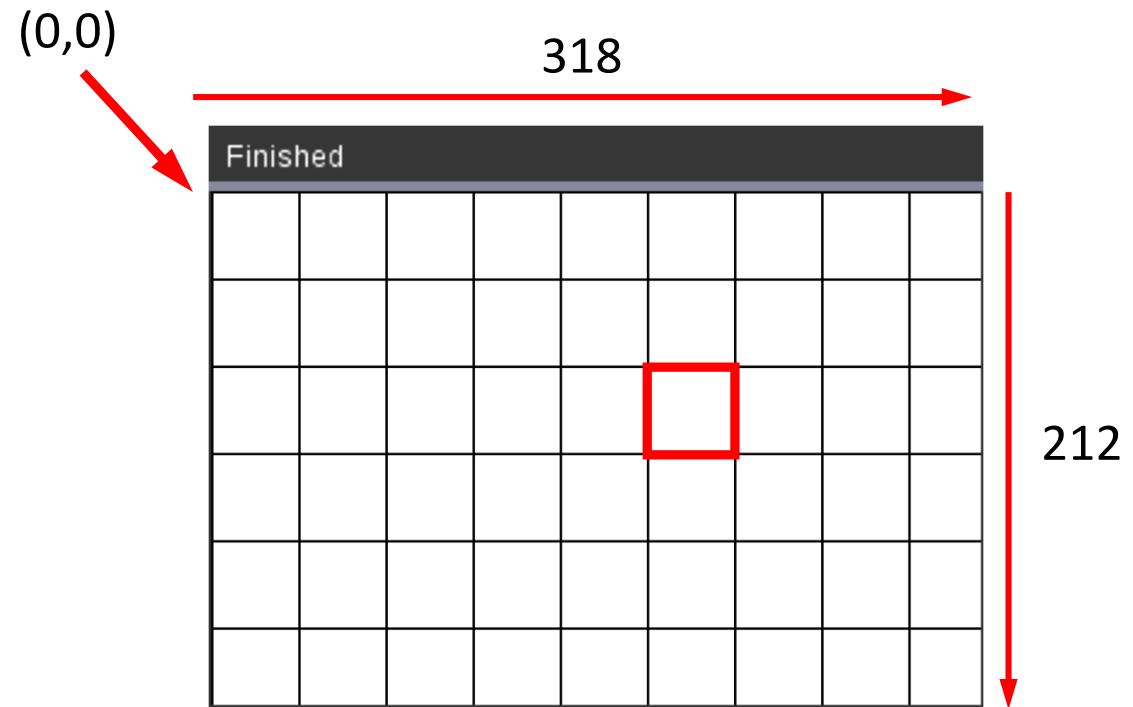


Regular tessellation

Rectangles



```
1.1 1.2 1.3 *01 Recta...les RAD [X]  
rectangles1.py 10/11  
from ti_draw import *  
  
a=36  
  
for i in range(6):  
    for j in range(9):  
        x=j*a  
        y=i*a  
        draw_rect(x,y,a,a)
```

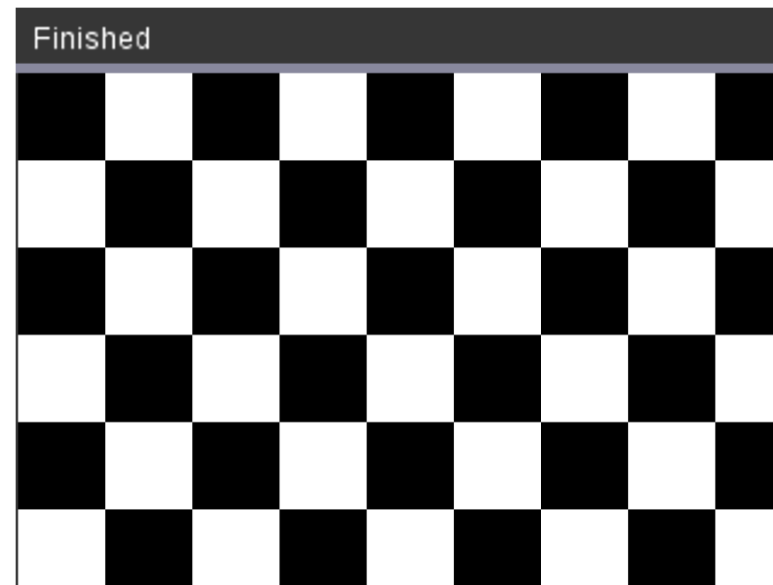
i : row number
j : column number





$$\begin{aligned} i &= 2 & x &= j \cdot 36 = 180 \\ j &= 5 & y &= i \cdot 36 = 72 \end{aligned}$$

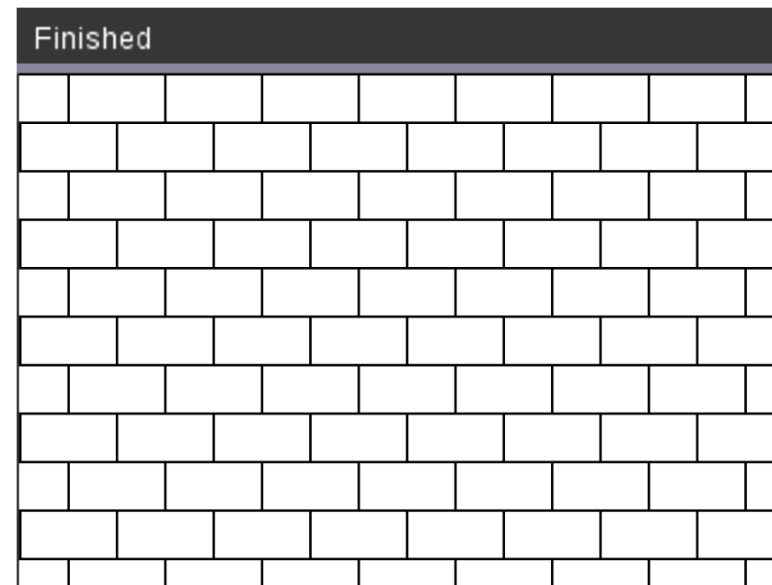
Rectangles

```
1.2 1.3 1.4 *01 Recta... les RAD    
rectangles2.py 2/11  
from ti_draw import *  
  
a=36  
  
for i in range(6):  
    for j in range(9):  
        x=j*a  
        y=i*a  
        if (i+j)%2 == 0:  
            fill_rect(x,y,a,a)
```





Rectangles

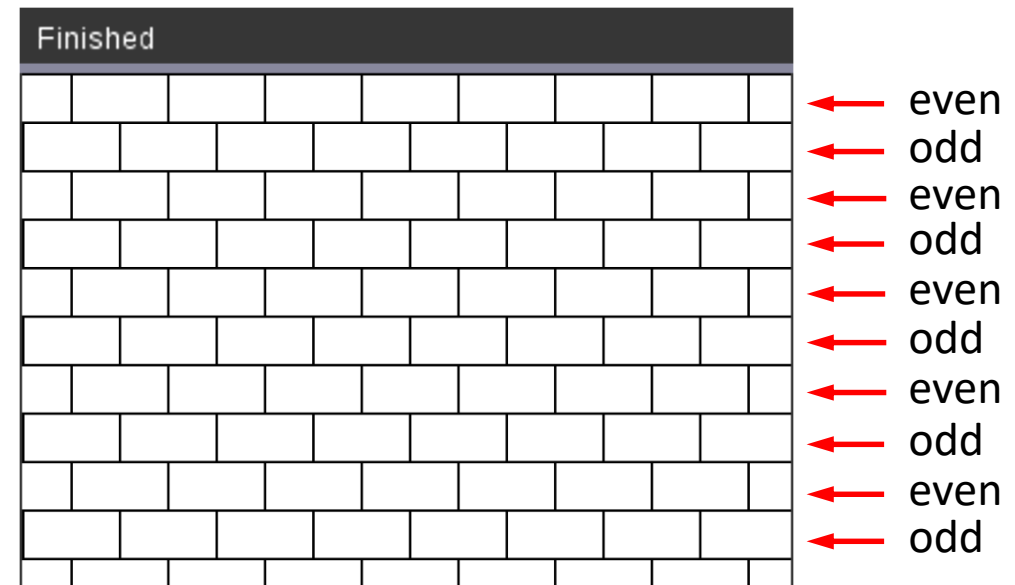
```
1.6 1.7 1.8 *01 Recta...les RAD    
rectangles4.py 2/12  
from ti_draw import *  
  
a=40  
b=20  
for i in range(11):  
    for j in range(9):  
        x=j*a-1  
        if i%2 == 0:  
            x = x-a/2  
        y=i*b  
        draw_rect(x,y,a,b)
```





p % q returns the remainder of p divided by q

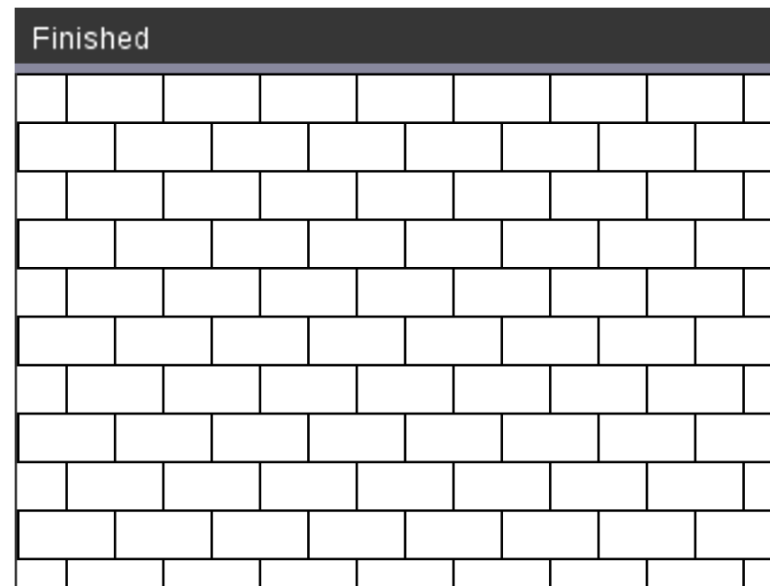
Rectangles

```
1.6 1.7 1.8 *01 Recta...les RAD    
rectangles4.py 2/12  
from ti_draw import *  
  
a=40  
b=20  
for i in range(11):  
    for j in range(9):  
        x=j*a-1  
        if i%2 == 0:  
            x = x-a/2  
        y=i*b  
        draw_rect(x,y,a,b)
```



Rectangles

```
1.8 1.9 1.10 *01 Recta...les RAD    
rectangles5.py 2/13  
a = 40  
b = 20  
→ use_buffer()  
for i in range(11):  
    ♦♦ for j in range(9):  
        ♦♦♦♦ x=j*a  
        ♦♦♦♦ if i%2 == 0:  
            ♦♦♦♦♦♦ x = x-a/2  
        ♦♦♦♦ y=i*b  
        ♦♦♦♦ draw_rect(x,y,a,b)  
→ paint_buffer()
```



Polygons

`draw_rect (x, y, width, height)`

`fill_rect (x, y, width, height)`

`draw_poly (xlist, ylist)`

`fill_poly (xlist, ylist)`

Polygons

`draw_rect (x, y, width, height)`

`fill_rect (x, y, width, height)`

`draw_poly (xlist, ylist)`

`fill_poly (xlist, ylist)`

Polygons

Module tessellations

multiply (*polygon*, *factor*)

rotate (*polygon*, *angle*)

mirror_hor (*polygon*)

mirror_vert (*polygon*)

draw (*x*, *y*, *polygon*)

fill (*x*, *y*, *polygon*)

x_correction (*polygon*)

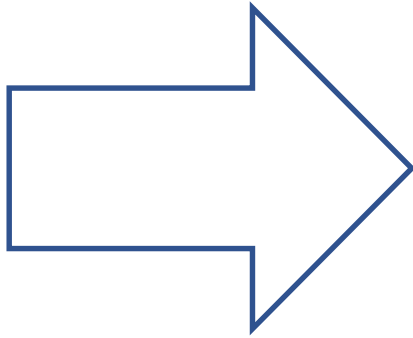
y_correction (*polygon*)

} *isometric grid*

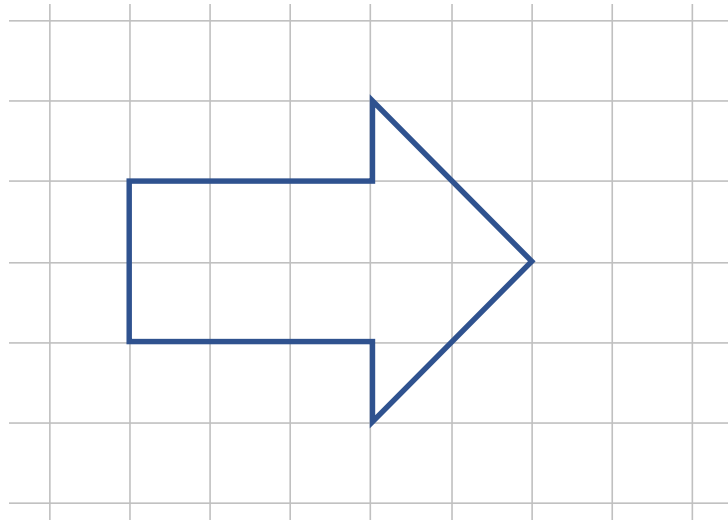
Polygon is a list with vertices

Window(-159,159,-106,106)

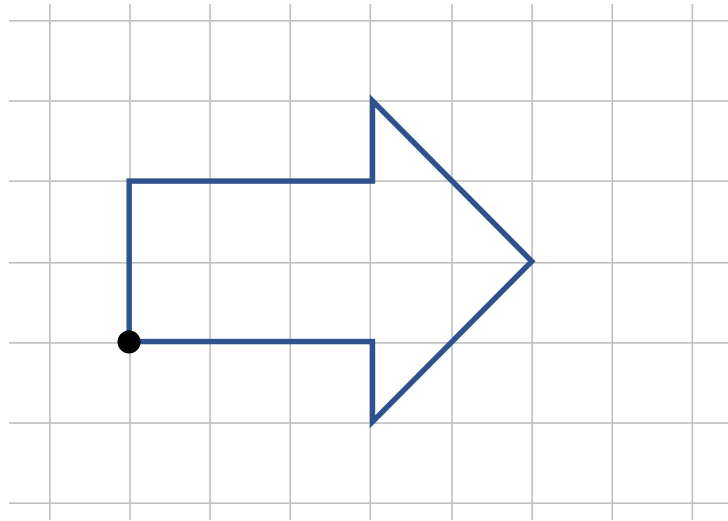
Polygons



Polygons

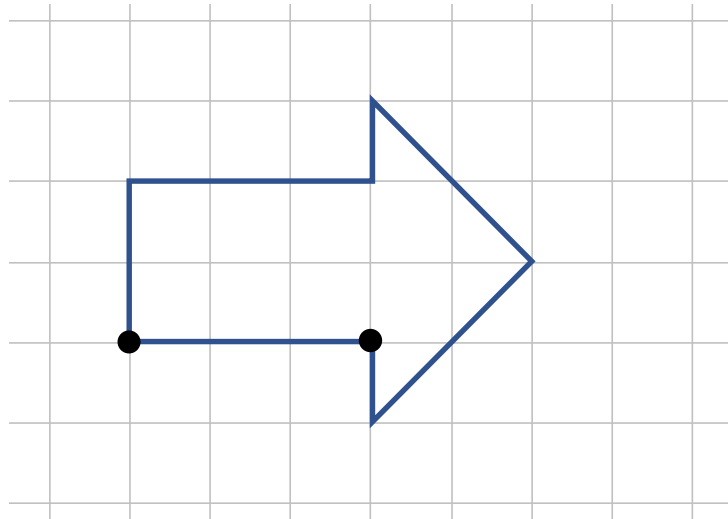


Polygons



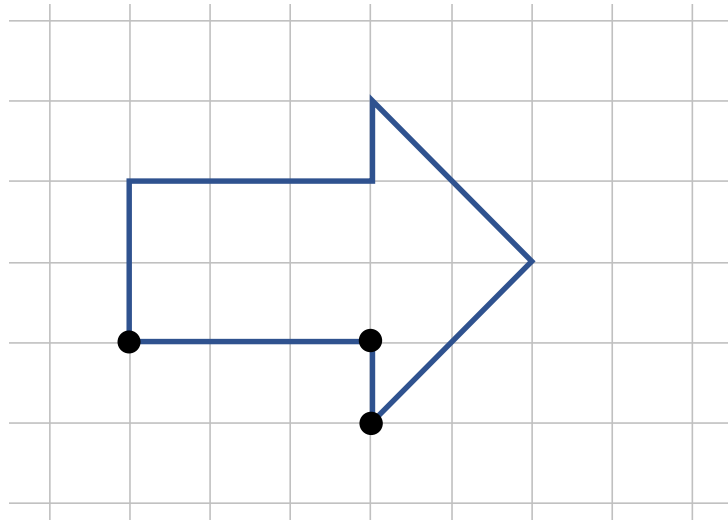
arrow = [(0,0)

Polygons



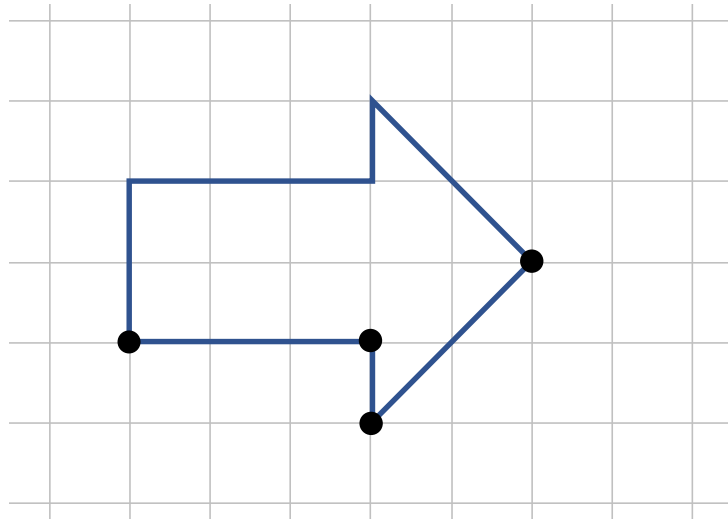
arrow = [(0,0), (3,0)]

Polygons



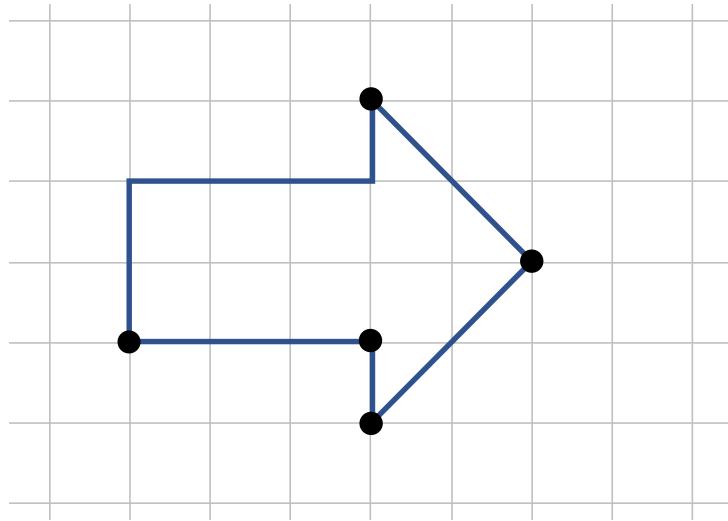
arrow = [(0,0), (3,0), (3,-1)]

Polygons



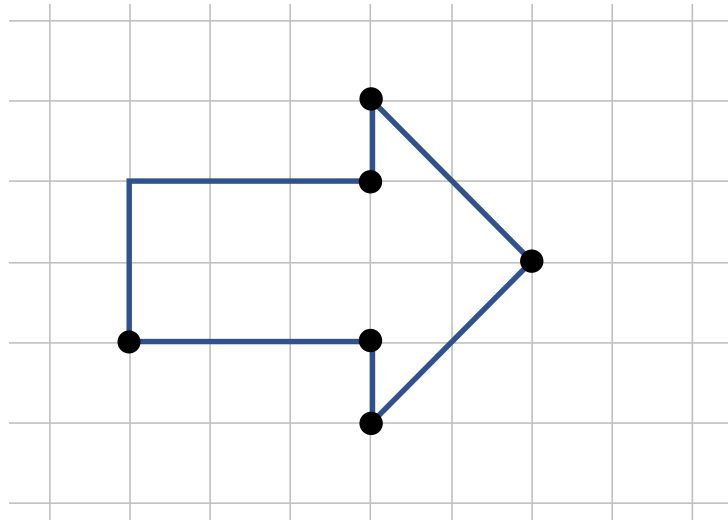
arrow = [(0,0), (3,0), (3,-1), (5,1)

Polygons



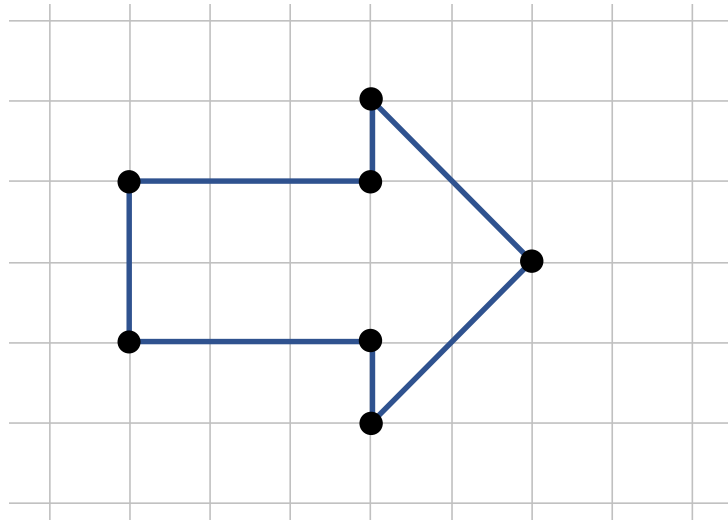
arrow = [(0,0), (3,0), (3, -1), (5,1), (3,3)

Polygons



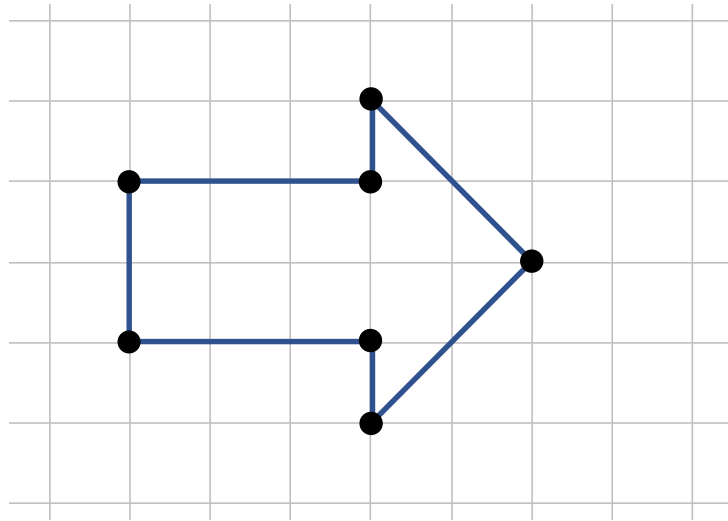
arrow = [(0,0), (3,0), (3, -1), (5,1), (3,3), (3,2)

Polygons



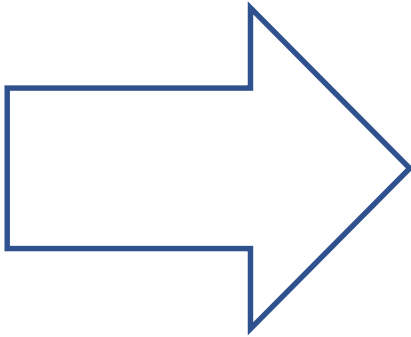
arrow = [(0,0), (3,0), (3, -1), (5,1), (3,3), (3,2), (0,2)

Polygons



arrow = [(0,0), (3,0), (3, -1), (5,1), (3,3), (3,2), (0,2), (0,0)]

Polygons



arrow = [(0,0), (3,0), (3, -1), (5,1), (3,3), (3,2), (0,2), (0,0)]

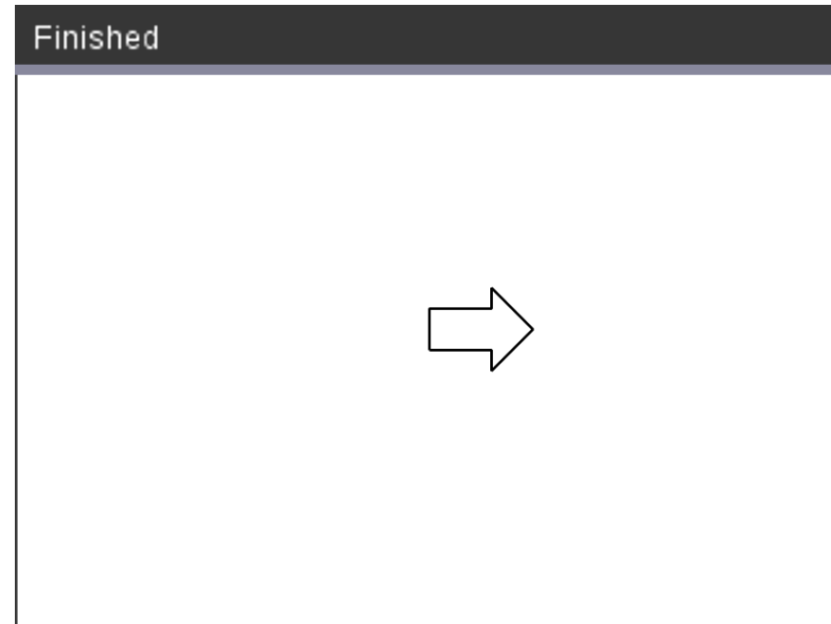
Polygons

```
1.1 1.2 1.3 *02 Arrows RAD 8/10
from tessellation import *

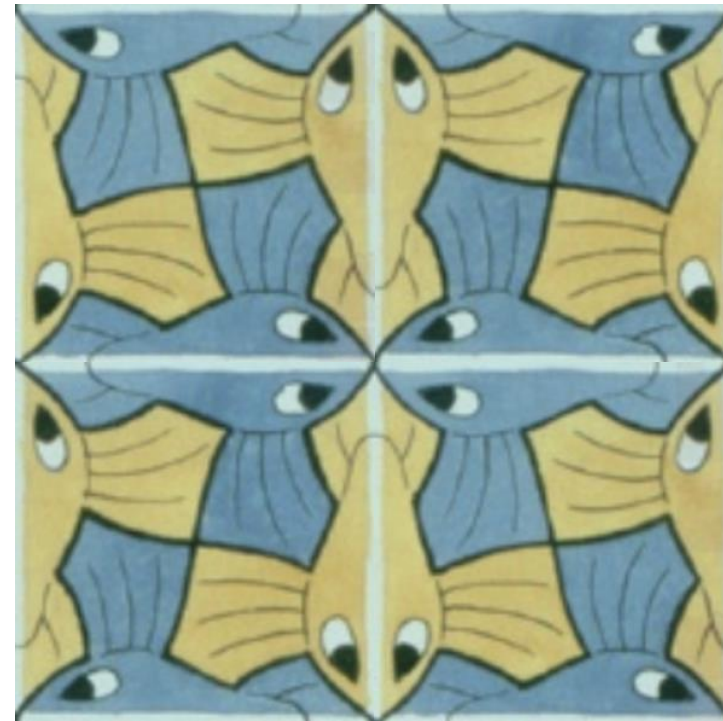
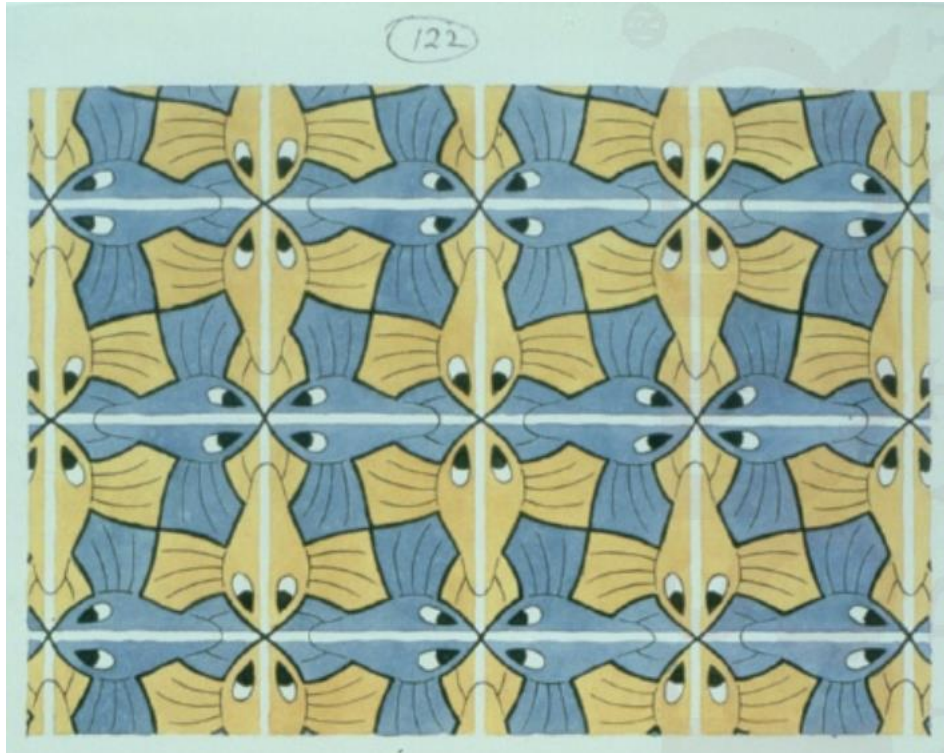
arrow = [
    (0,0), (3,0), (3,-1), (5,1),
    (3,3), (3,2), (0,2), (0,0)]

arrow = multiply(arrow, 8)

draw(0, 0, arrow)
```

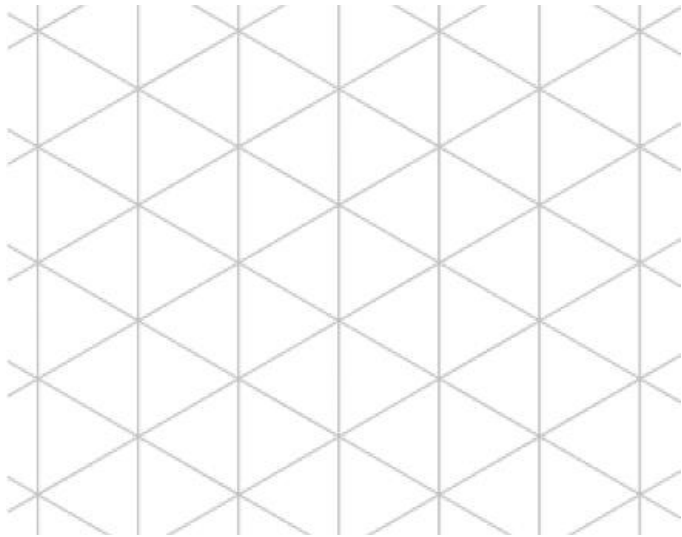


Polygons

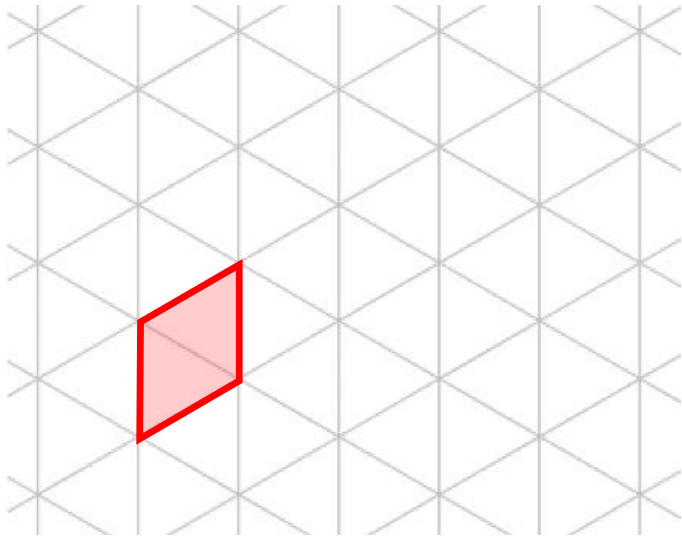


Example 1

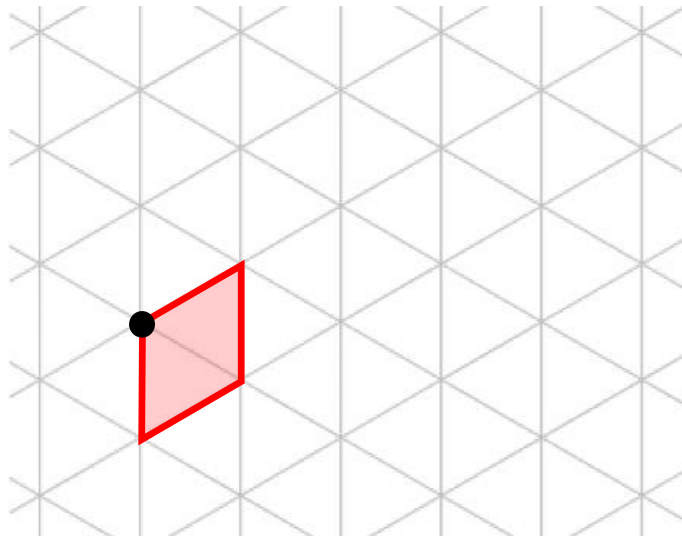
Isometric grid



Isometric grid

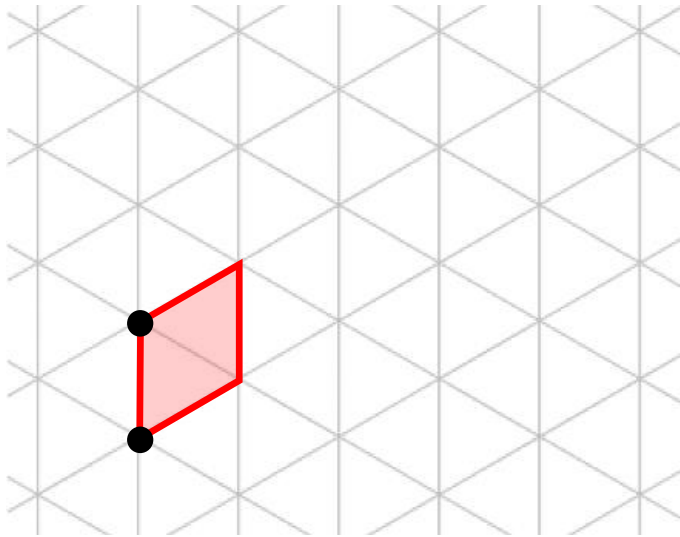


Isometric grid



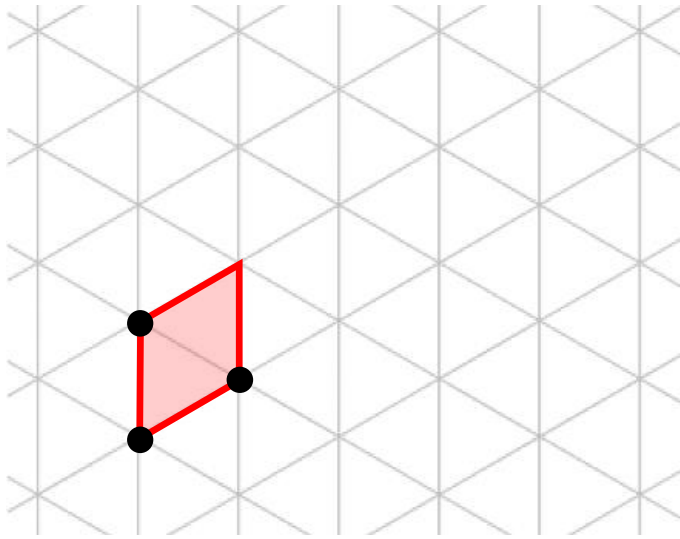
kite = [(0, 0)

Isometric grid



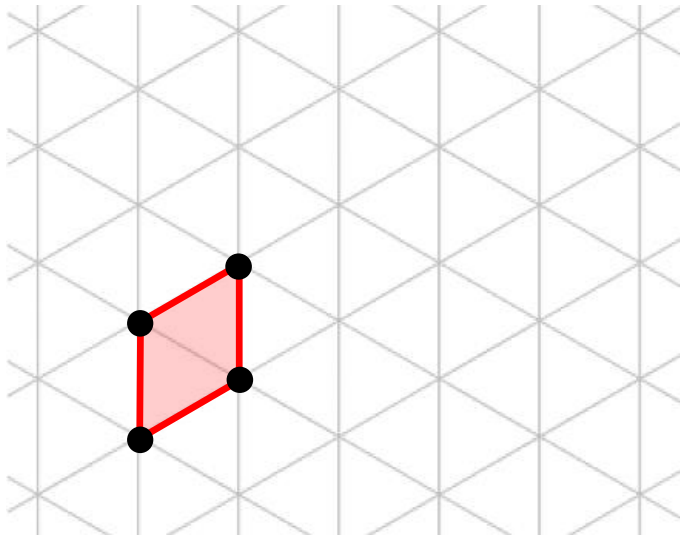
kite = [(0, 0), (0, -1)

Isometric grid



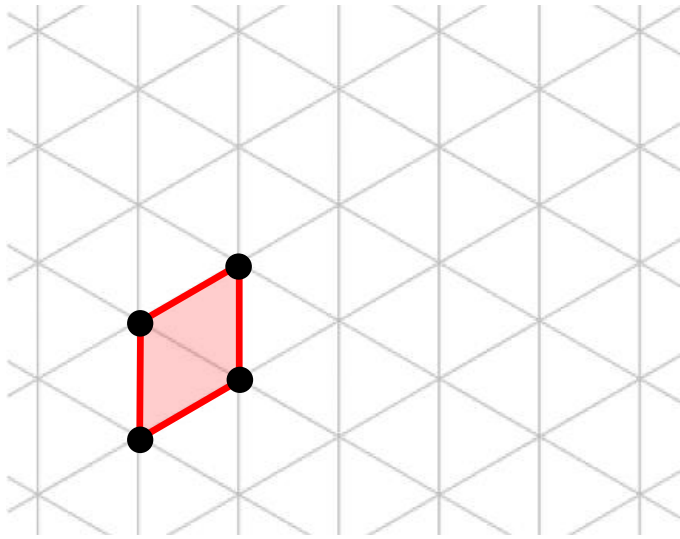
$$\text{kite} = [(0, 0), (0, -1), (\frac{1}{2}\sqrt{3}, -\frac{1}{2})]$$

Isometric grid



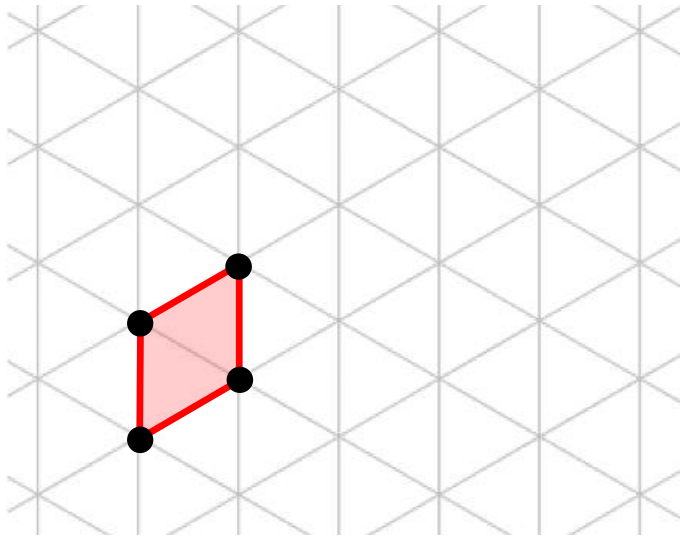
$$\text{kite} = [(0, 0), (0, -1), (\frac{1}{2}\sqrt{3}, -\frac{1}{2}), (\frac{1}{2}\sqrt{3}, \frac{1}{2})]$$

Isometric grid



$$\text{kite} = [(0, 0), (0, -1), (\frac{1}{2}\sqrt{3}, -\frac{1}{2}), (\frac{1}{2}\sqrt{3}, \frac{1}{2}), (0, 0)]$$

Isometric grid

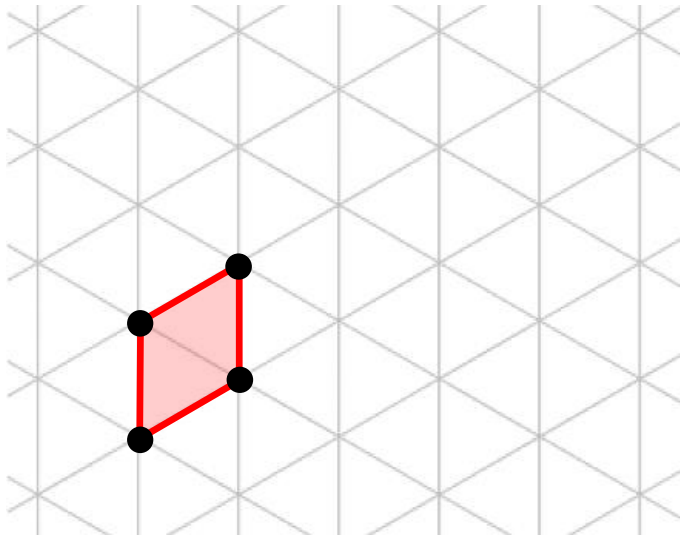


$$\text{kite} = [(0, 0), (0, -1), (1, -\frac{1}{2}), (1, \frac{1}{2}), (0, 0)]$$

$$\text{kite} = \text{x_correction}(\text{kite})$$

$$\text{kite} = [(0, 0), (0, -1), (\frac{1}{2}\sqrt{3}, -\frac{1}{2}), (\frac{1}{2}\sqrt{3}, \frac{1}{2}), (0, 0)]$$

Isometric grid



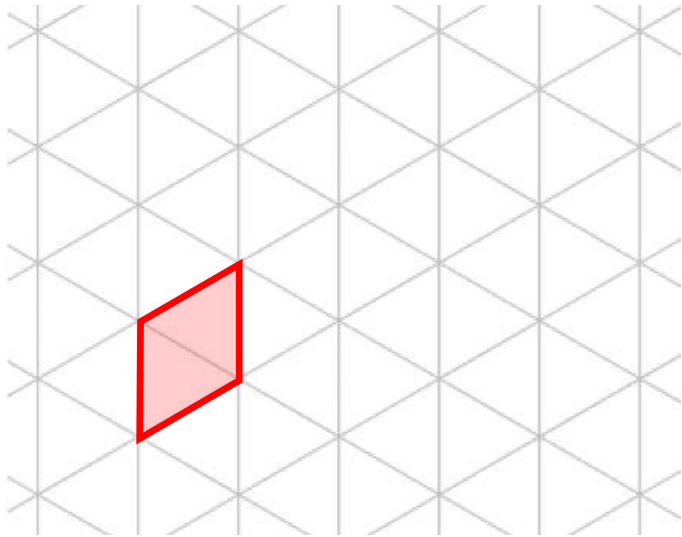
$$\text{kite} = [(0, 0), (0, -1), (1, -\frac{1}{2}), (1, \frac{1}{2}), (0, 0)]$$



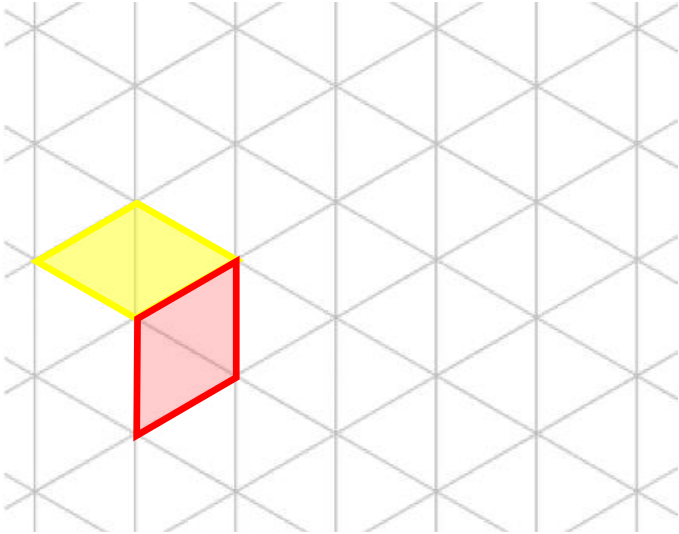
$$\text{kite} = \text{x_correction}(\text{kite})$$

$$\text{kite} = [(0, 0), (0, -1), (\frac{1}{2}\sqrt{3}, -\frac{1}{2}), (\frac{1}{2}\sqrt{3}, \frac{1}{2}), (0, 0)]$$

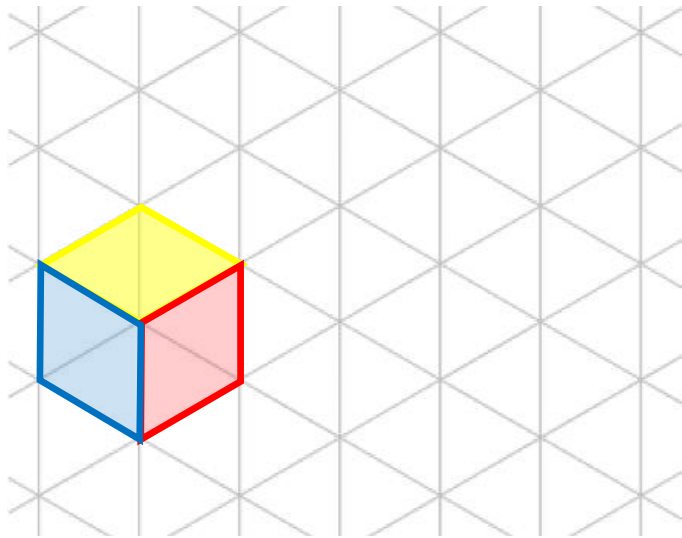
Isometric grid



Isometric grid



Isometric grid

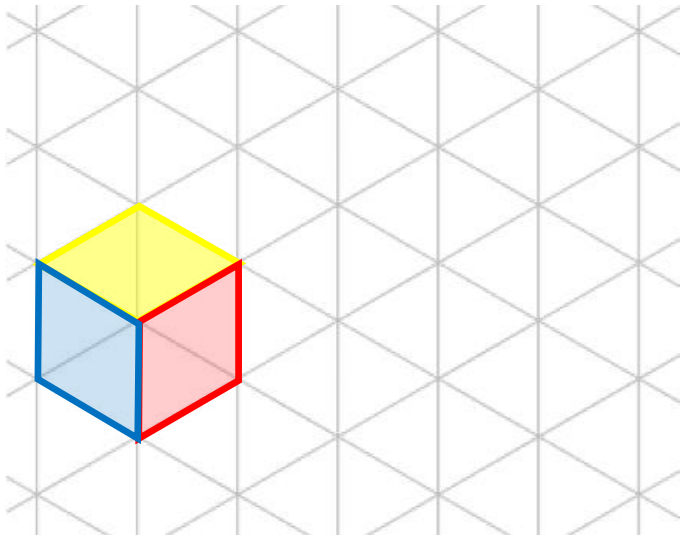


```
kite = [(0,0), (0,-1), (1,-0.5), (1,0.5), (0,0)]  
kite = x_correction(kite)
```

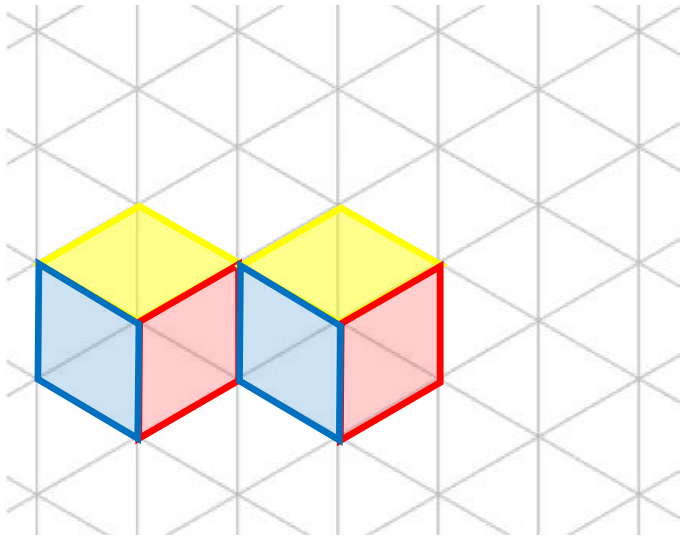
```
colors = [(255,0,0), (255,255,0), (0,0,255)]
```

```
def hexagon(x,y):  
    for i in range(3):  
        kyte = rotate(kite, 120*i)  
        set_color(colors[i])  
        fill(x,y,kyte)
```

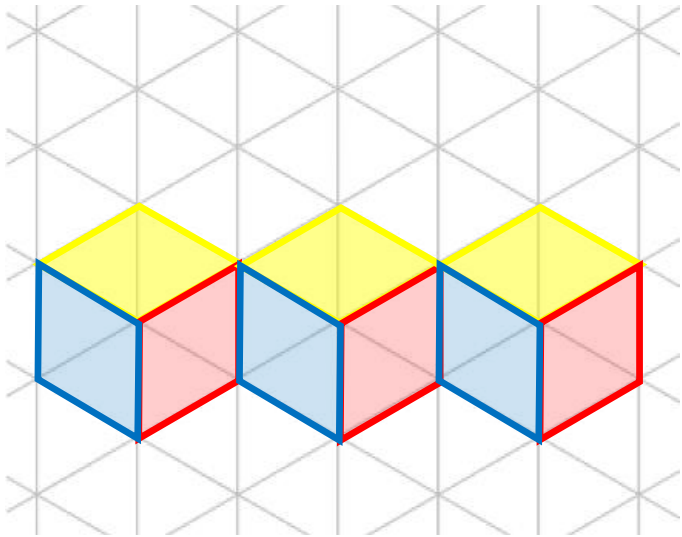

Isometric grid



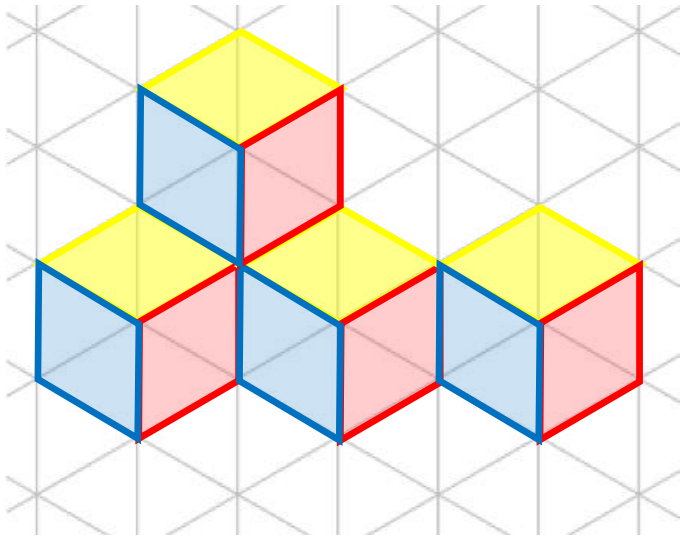
Isometric grid



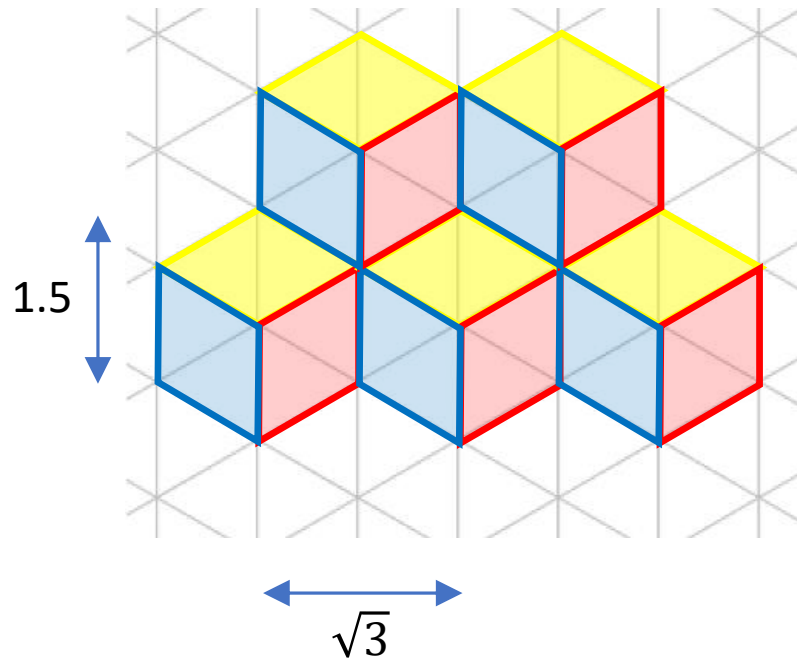
Isometric grid



Isometric grid



Isometric grid

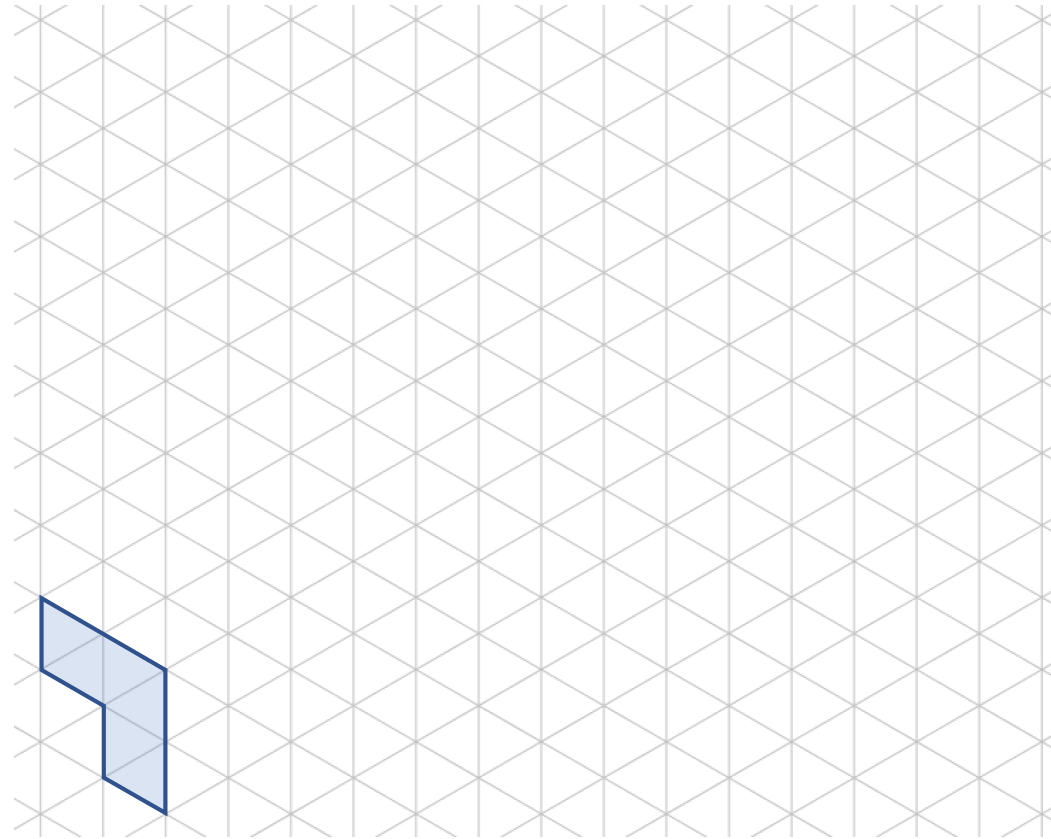


```
for i in range(-2,3):  
    for j in range(-3,4):  
        x = j*sqrt(3)*a  
        if i%2 == 0:  
            x = x+0.5*sqrt(3)*a  
        y = i*1.5*a  
        hexagon(x,y)
```

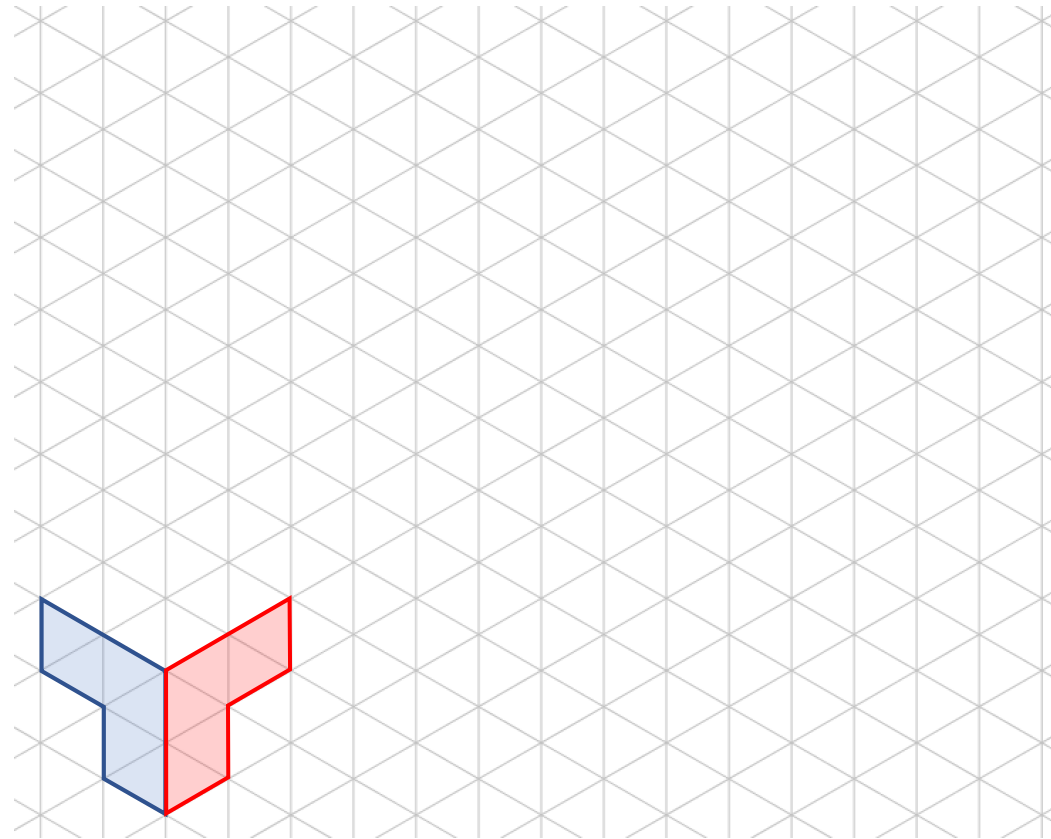
T-shapes



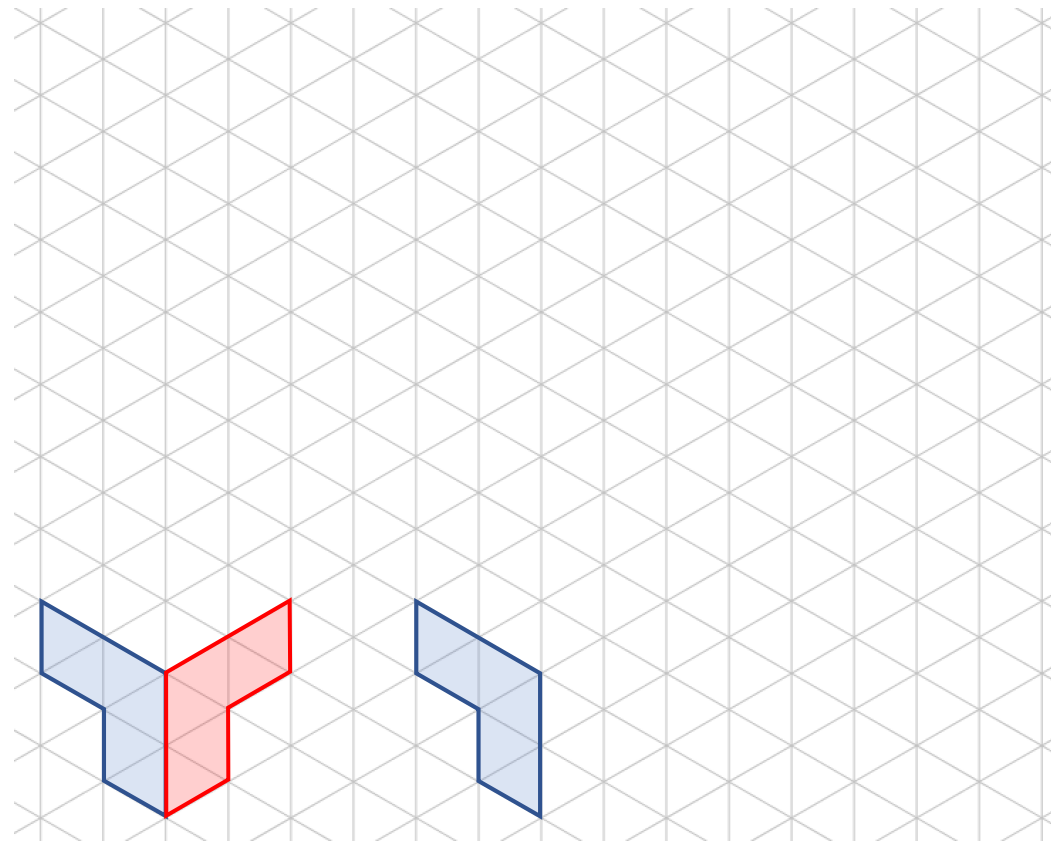
T-shapes



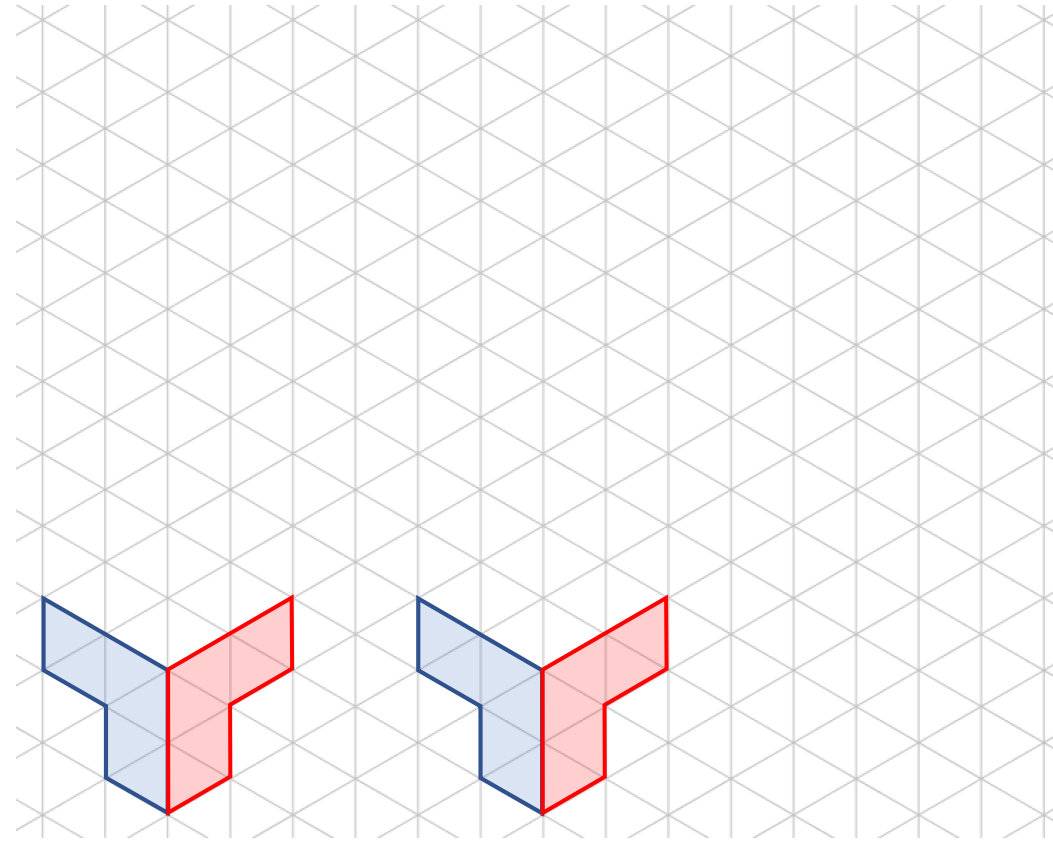
T-shapes



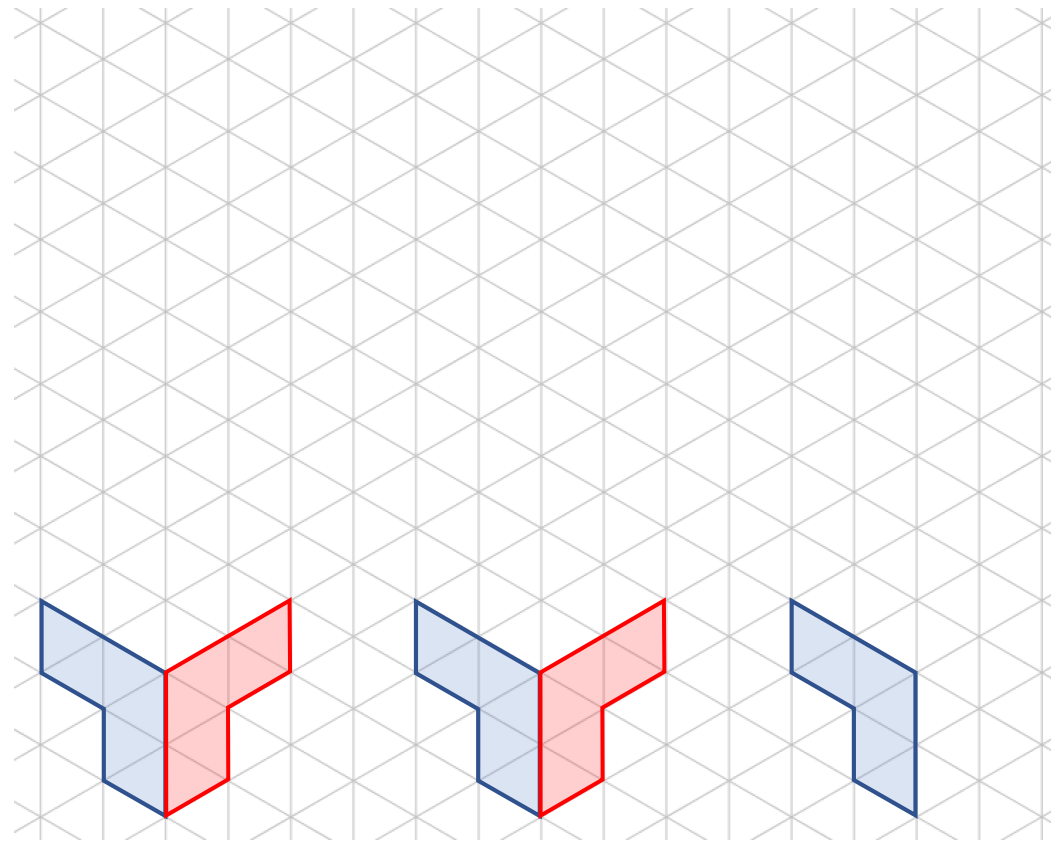
T-shapes



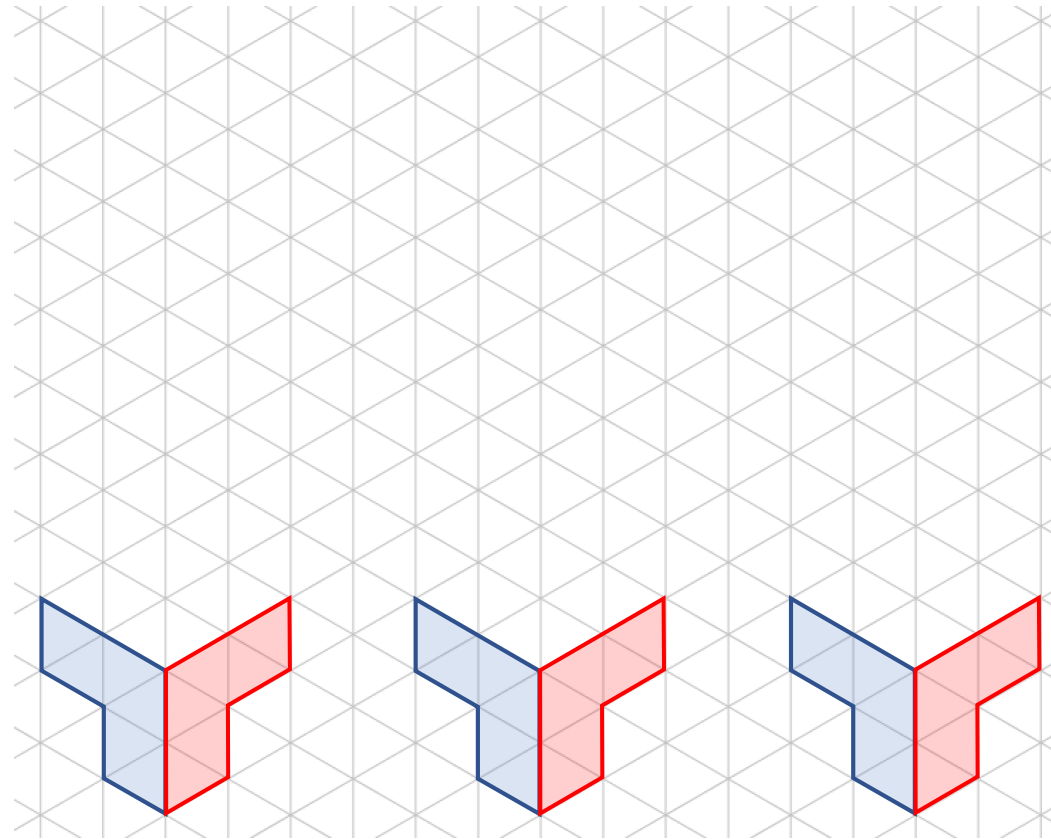
T-shapes



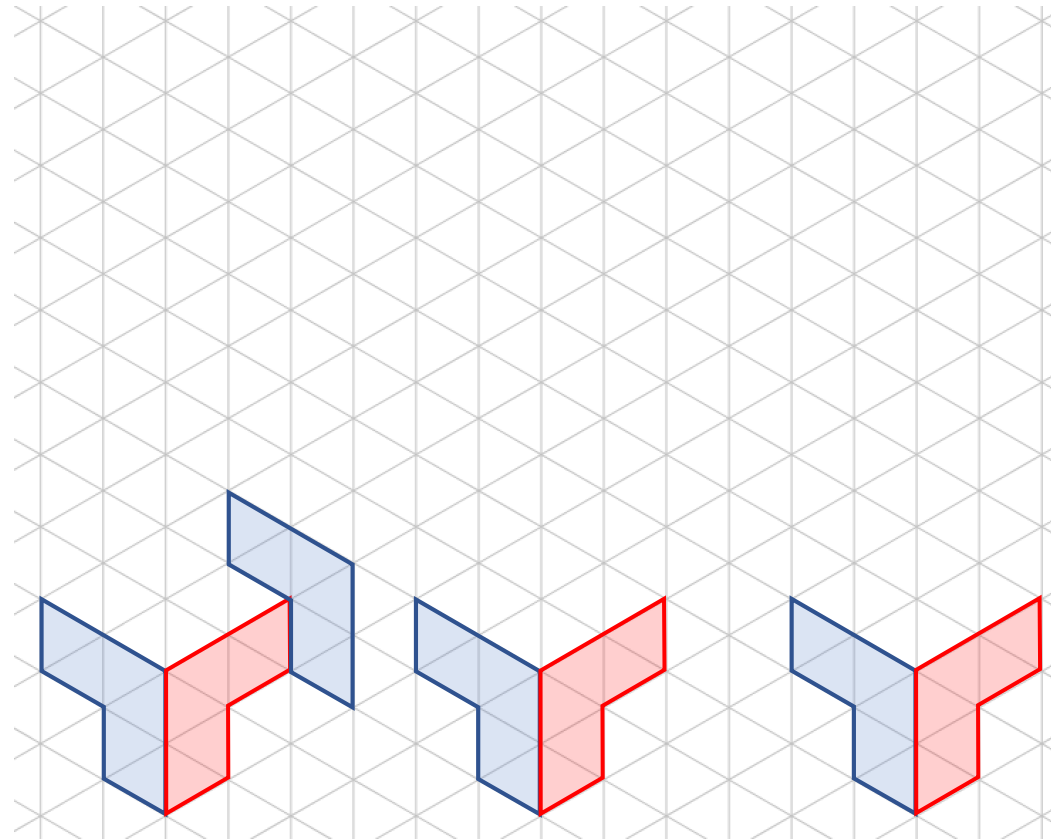
T-shapes



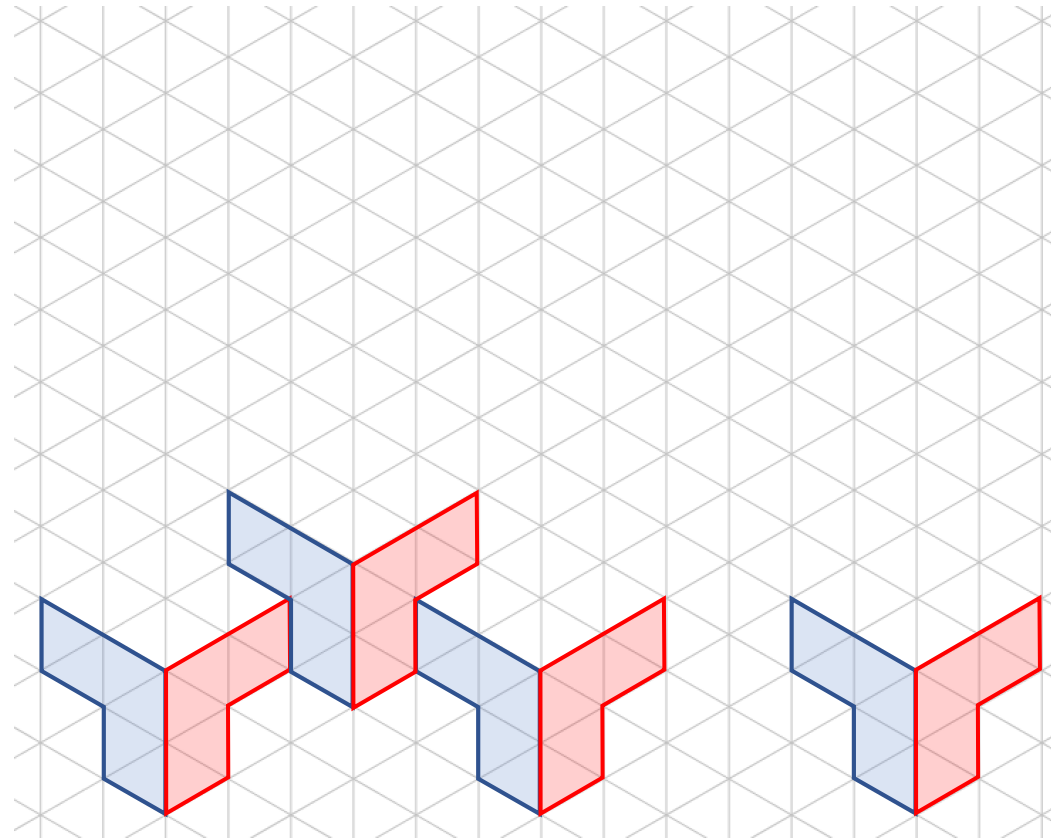
T-shapes



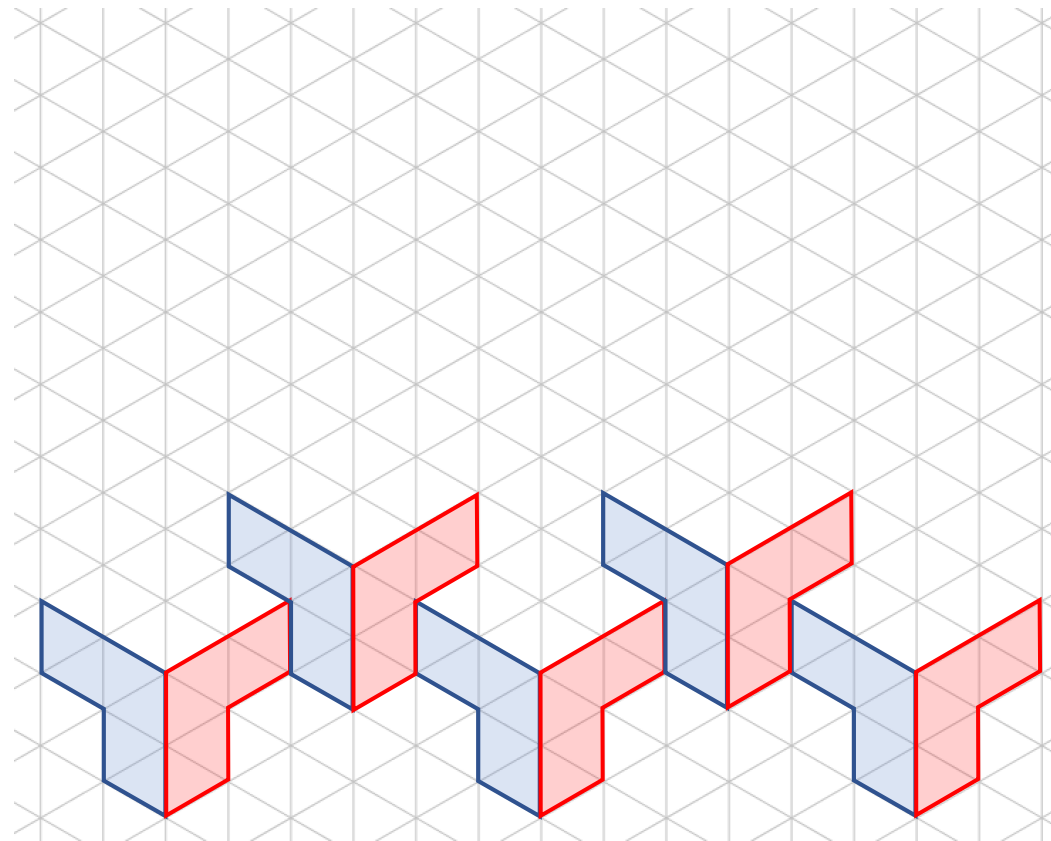
T-shapes



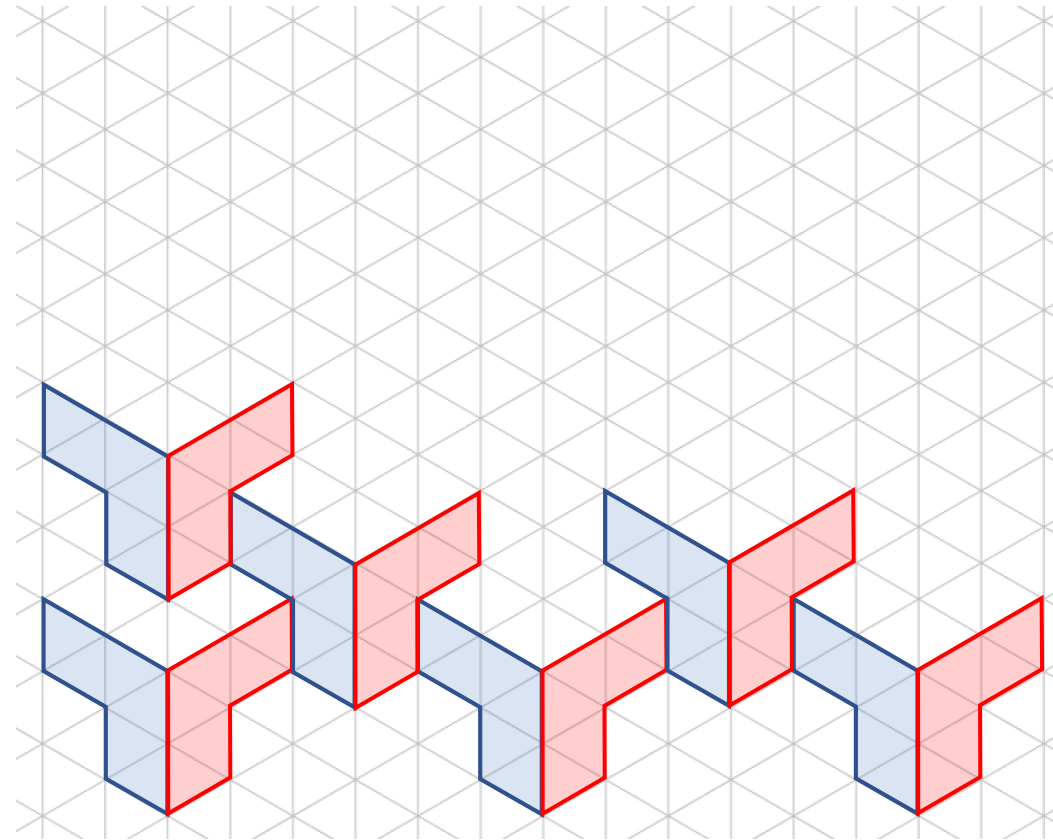
T-shapes



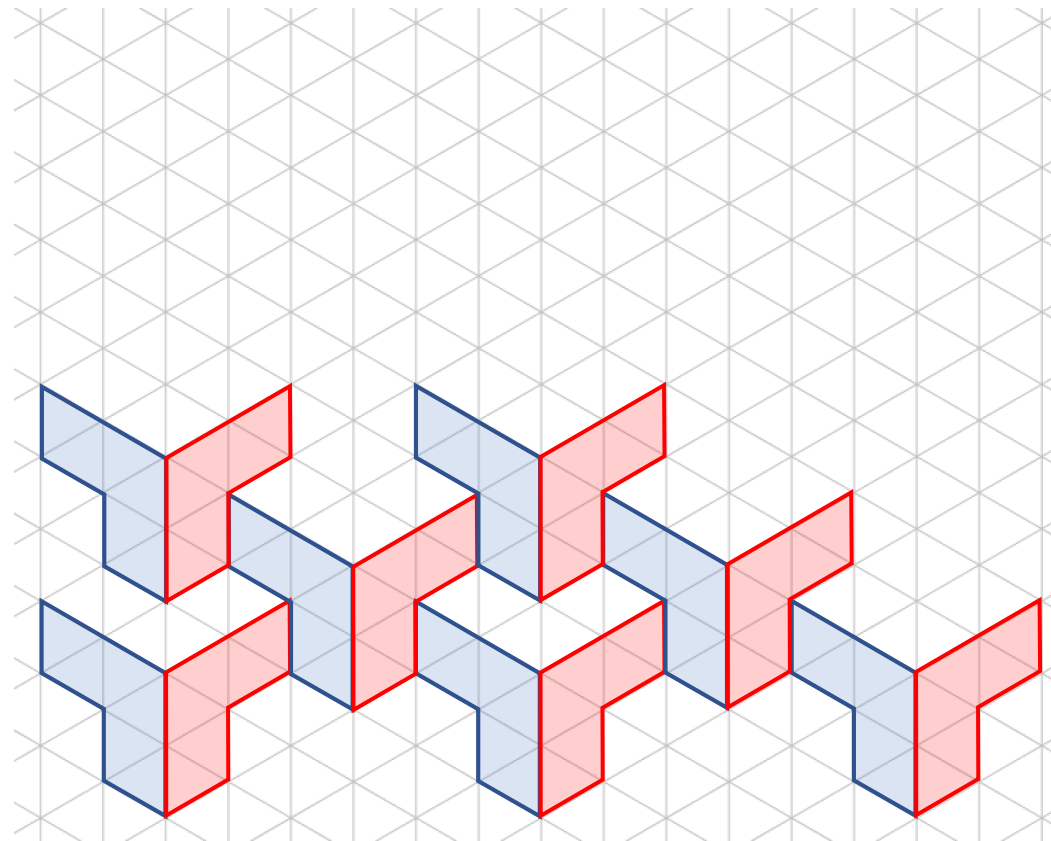
T-shapes



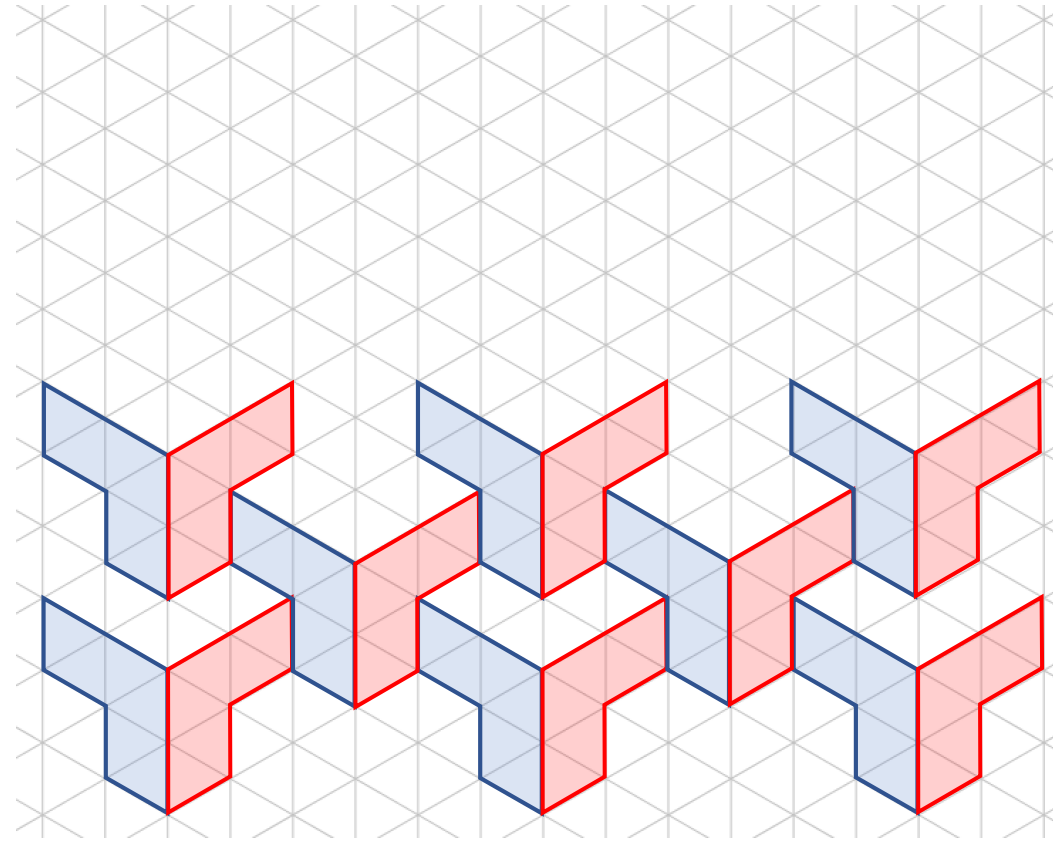
T-shapes



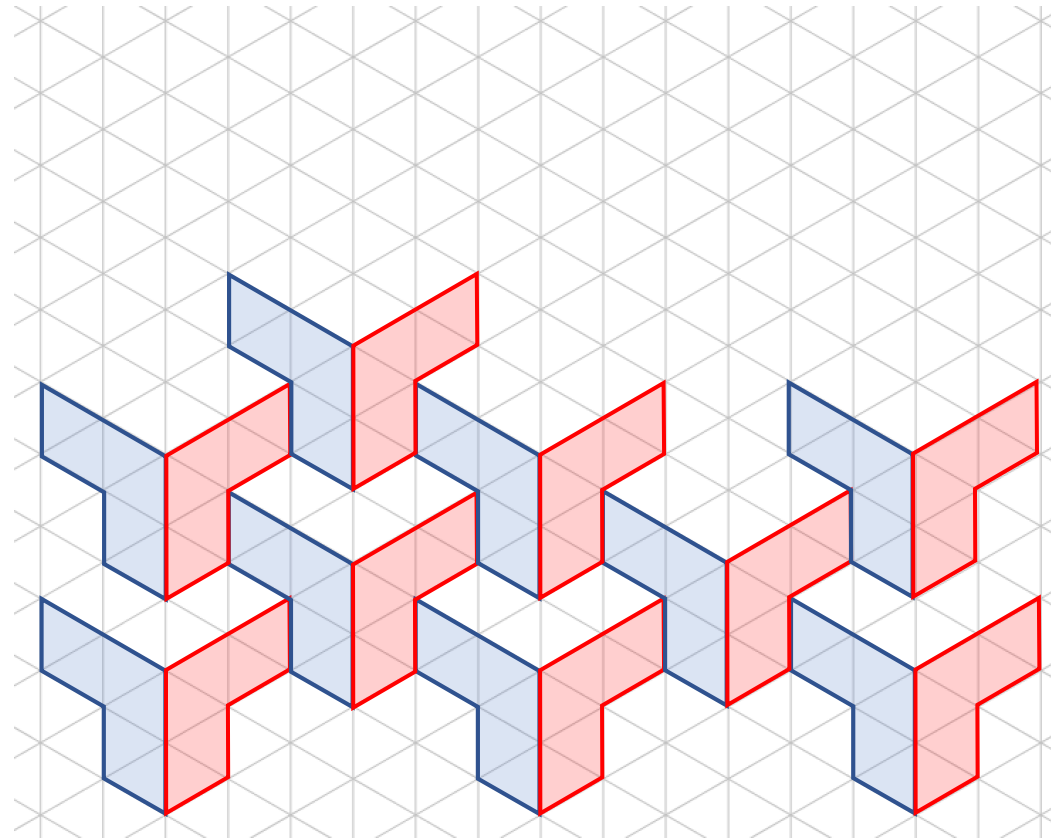
T-shapes



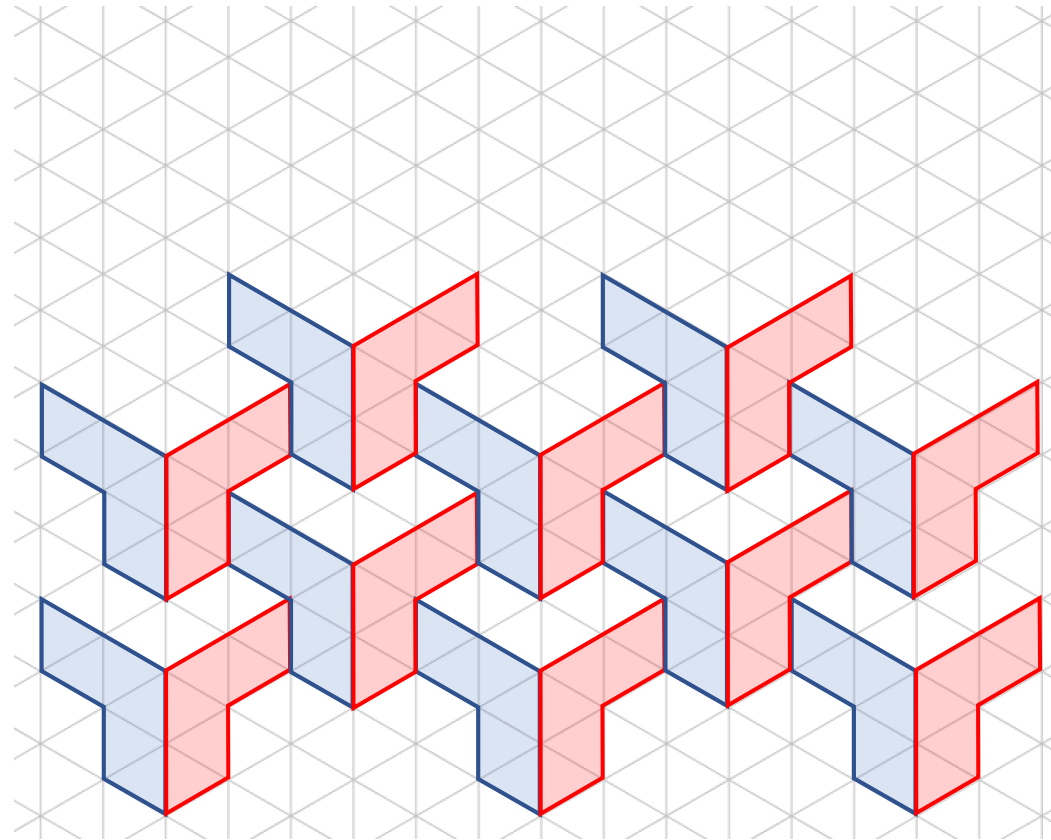
T-shapes



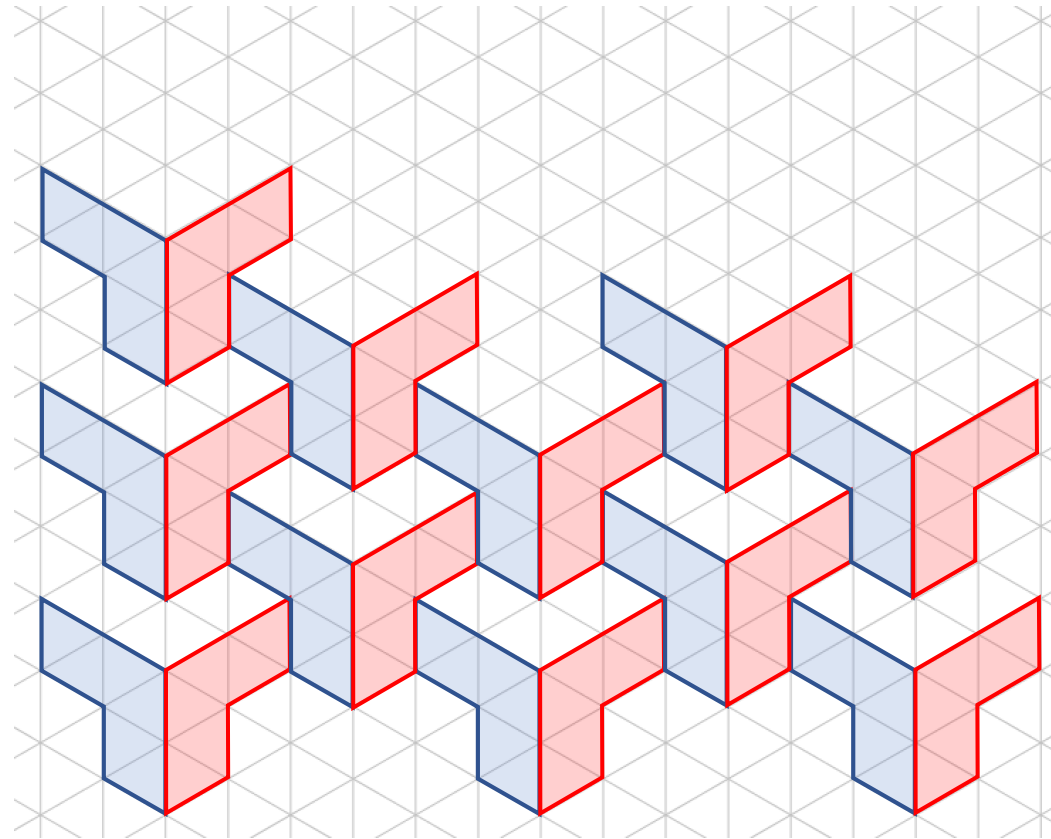
T-shapes



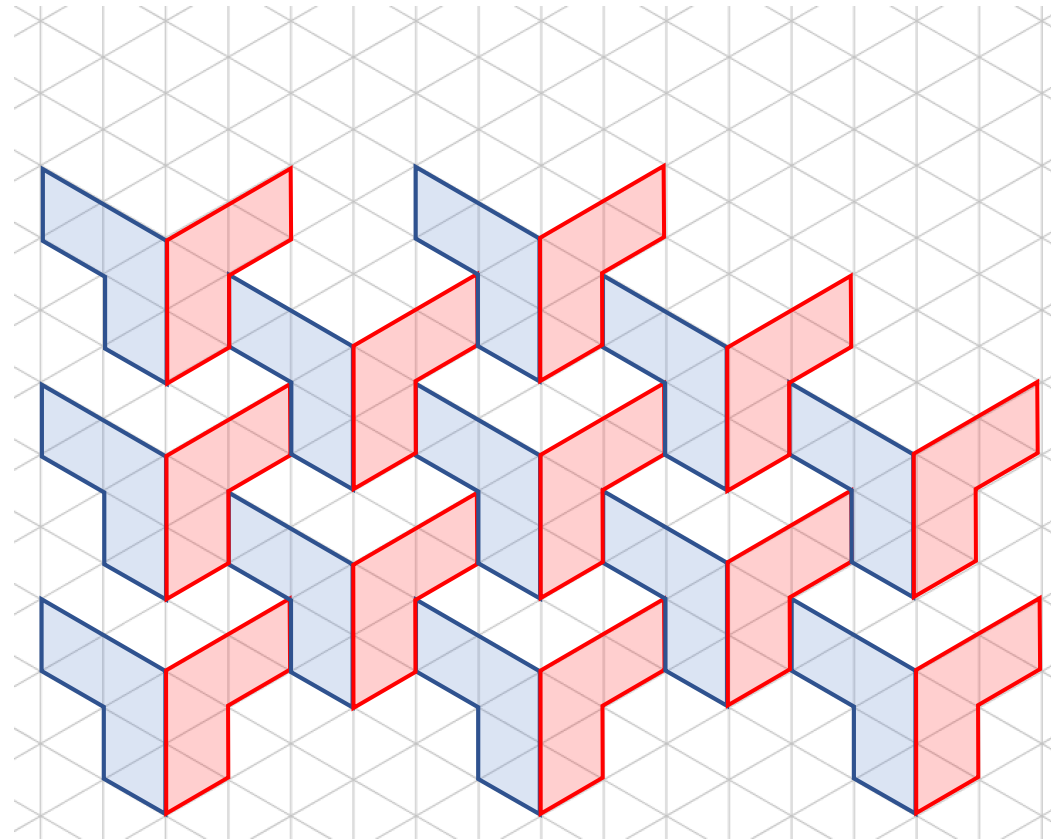
T-shapes



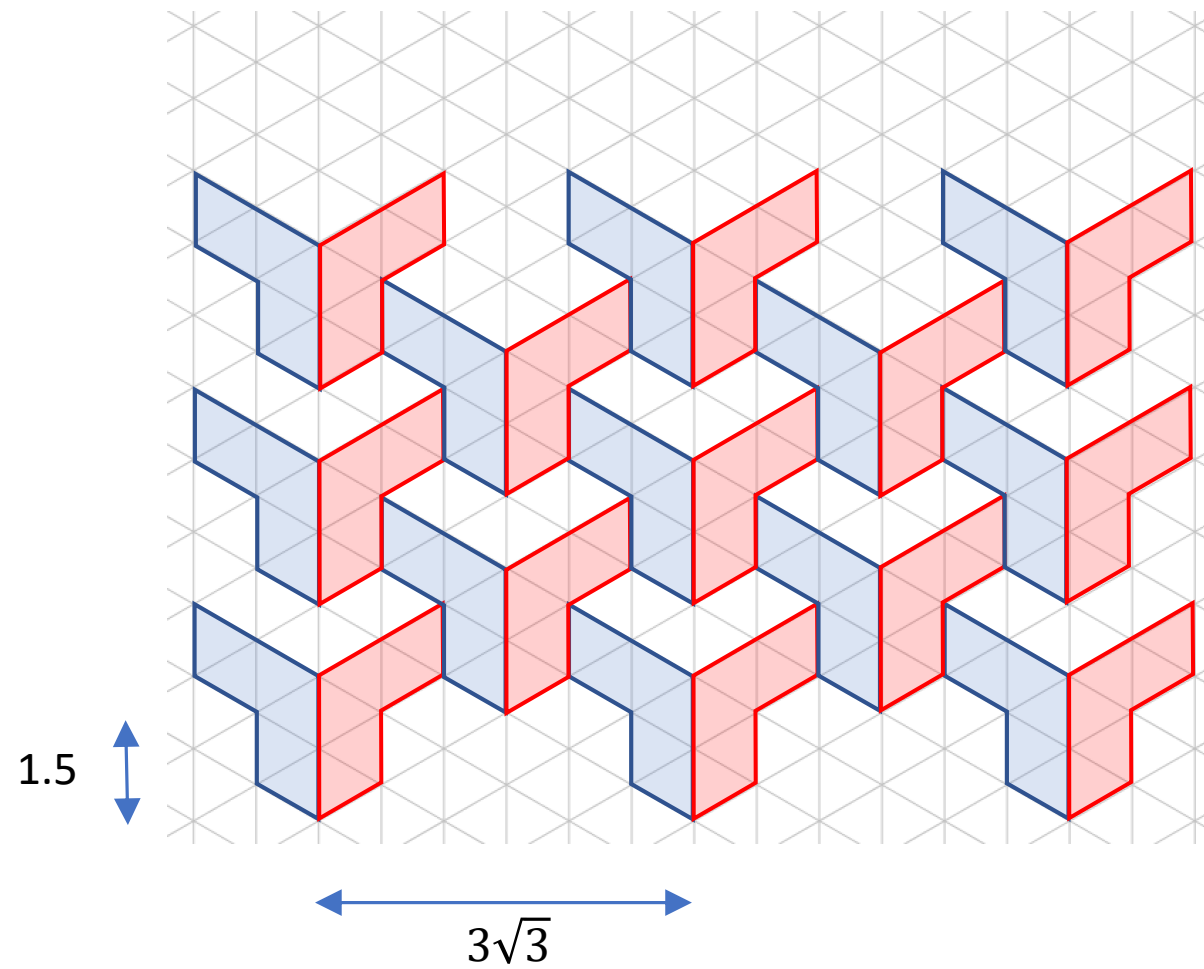
T-shapes



T-shapes



T-shapes



Tessellations

Tessellations

Cairo

Fish

Circular Fish

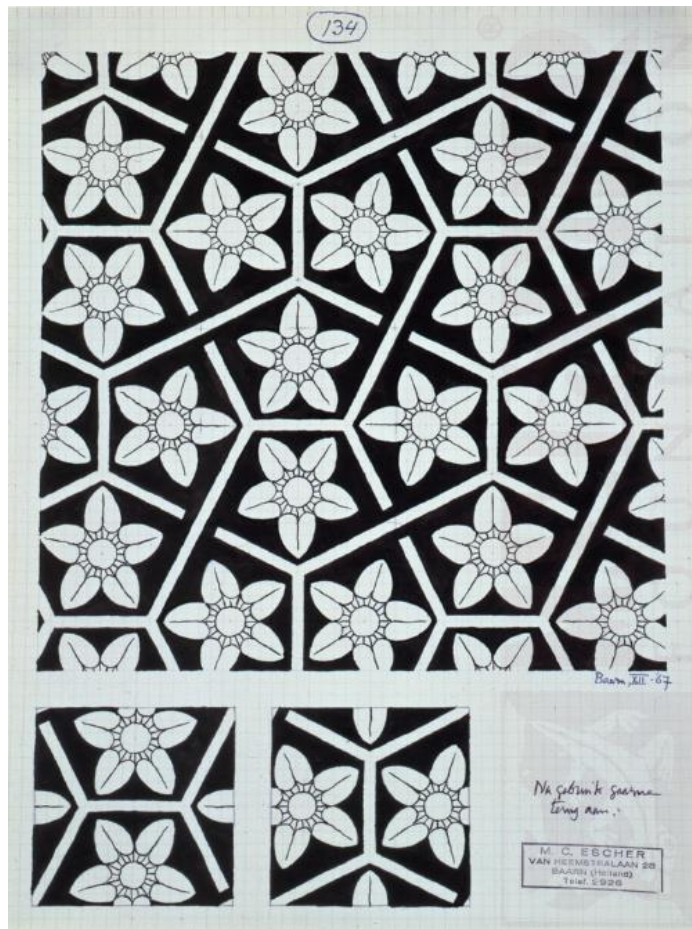
Penrose

End

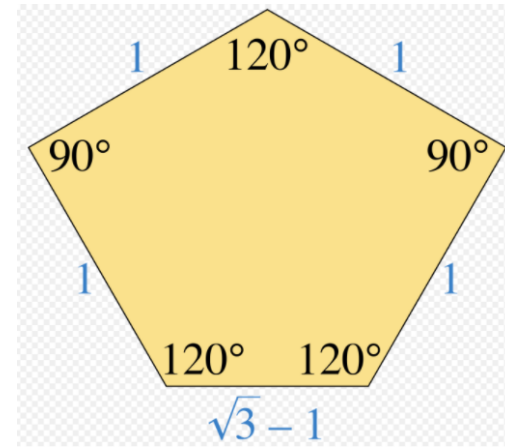
Cairo



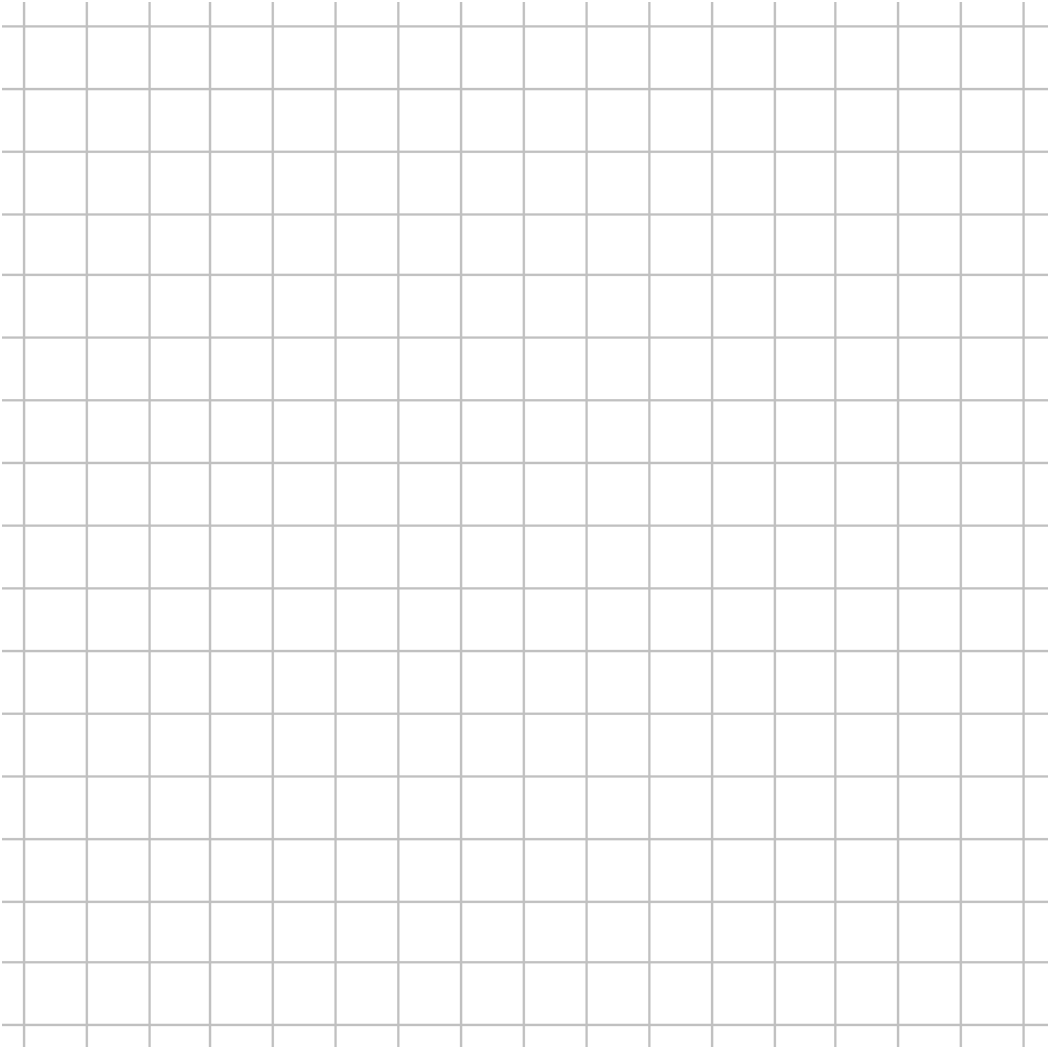
Cairo



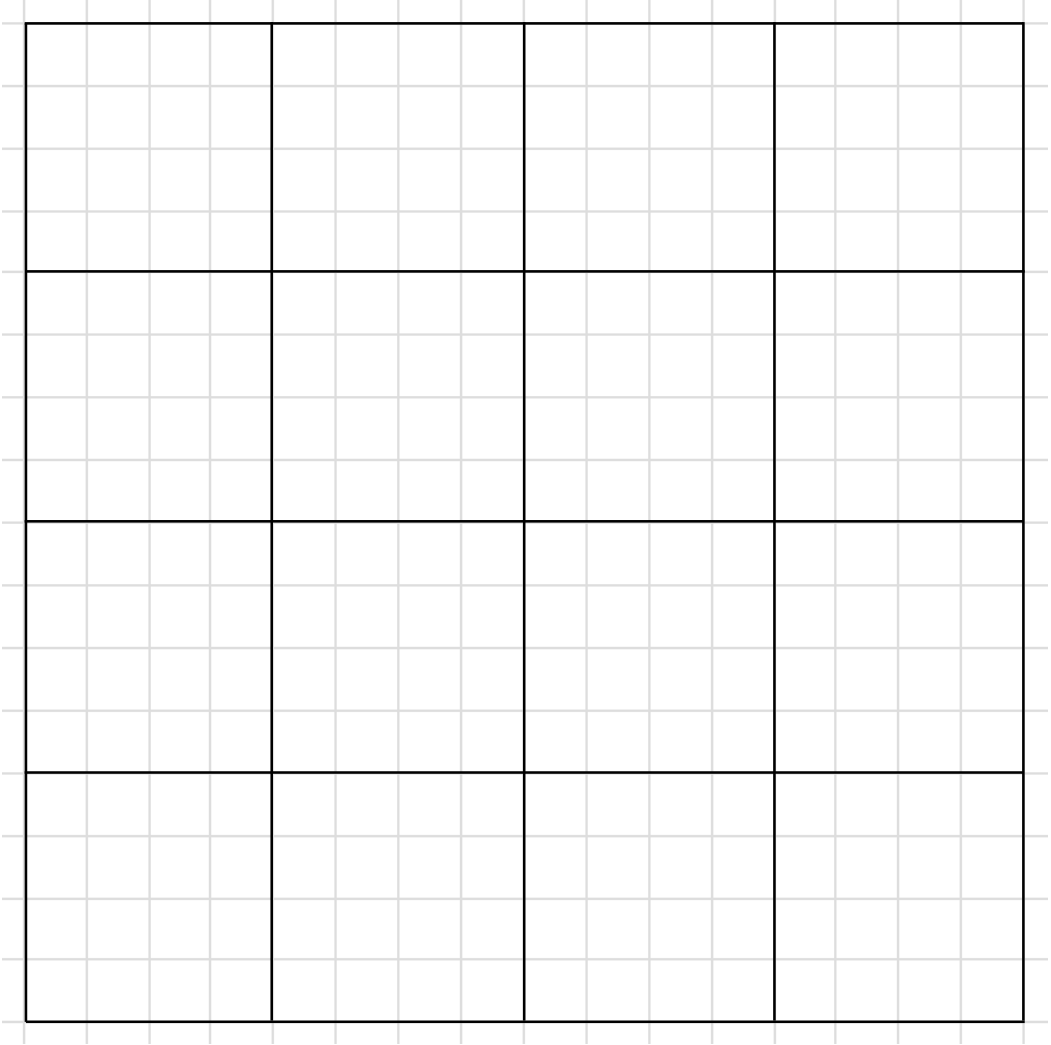
Cairo



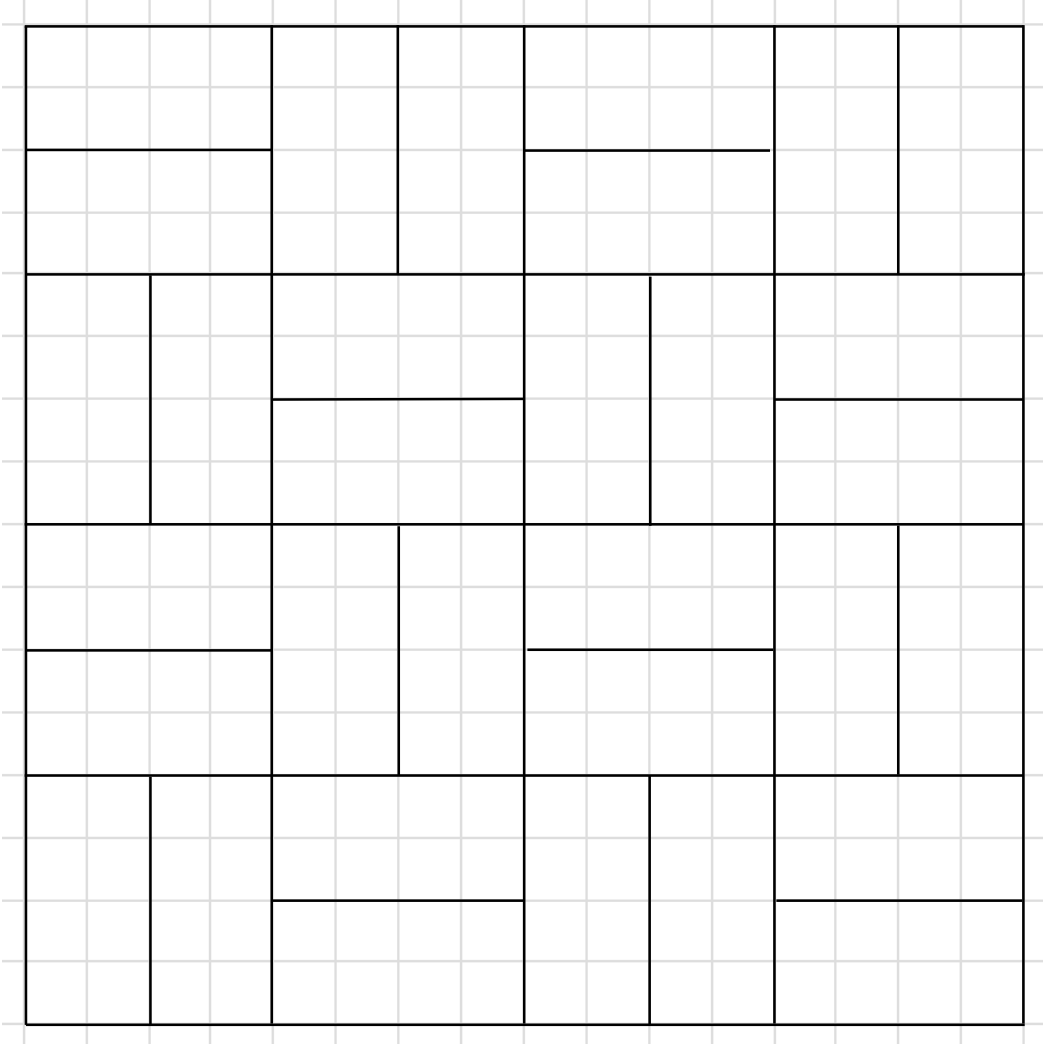
Cairo



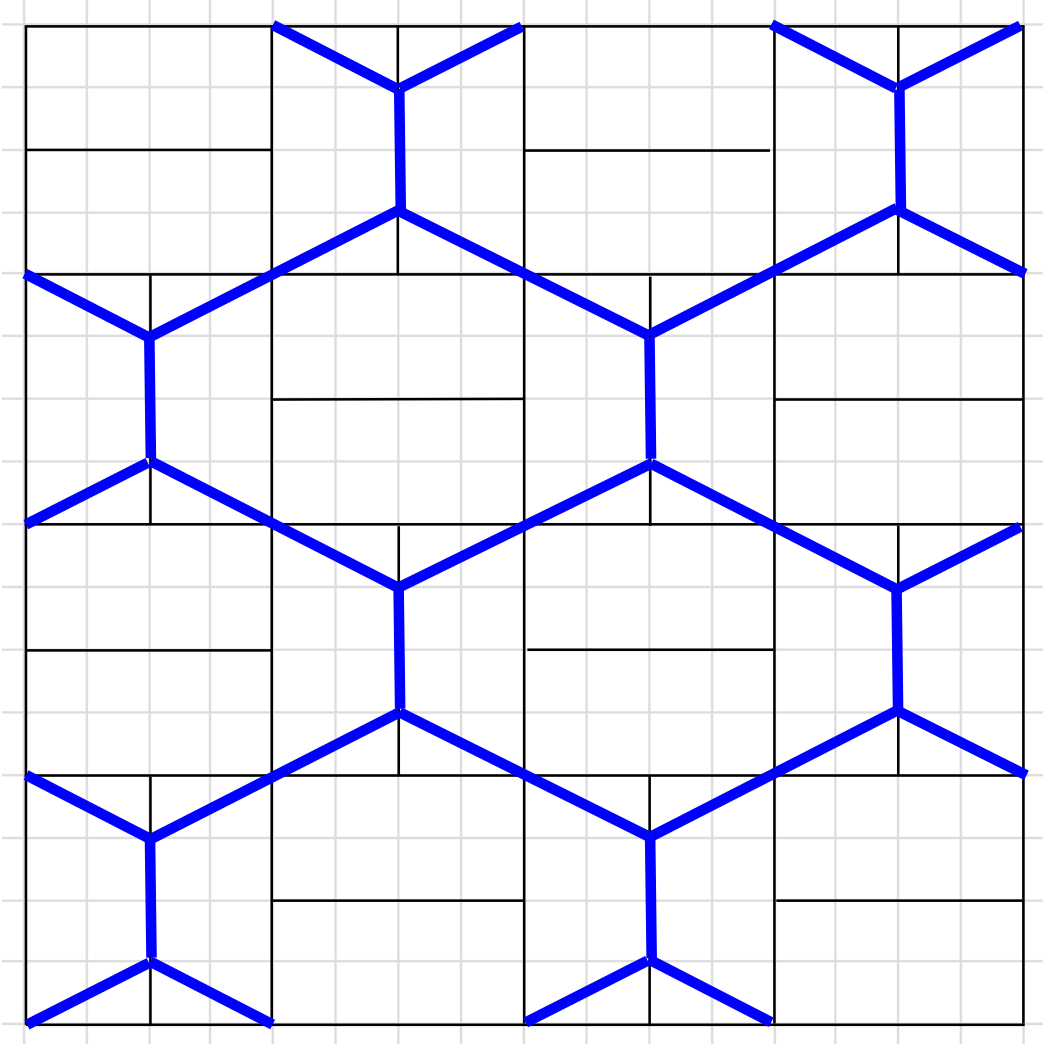
Cairo



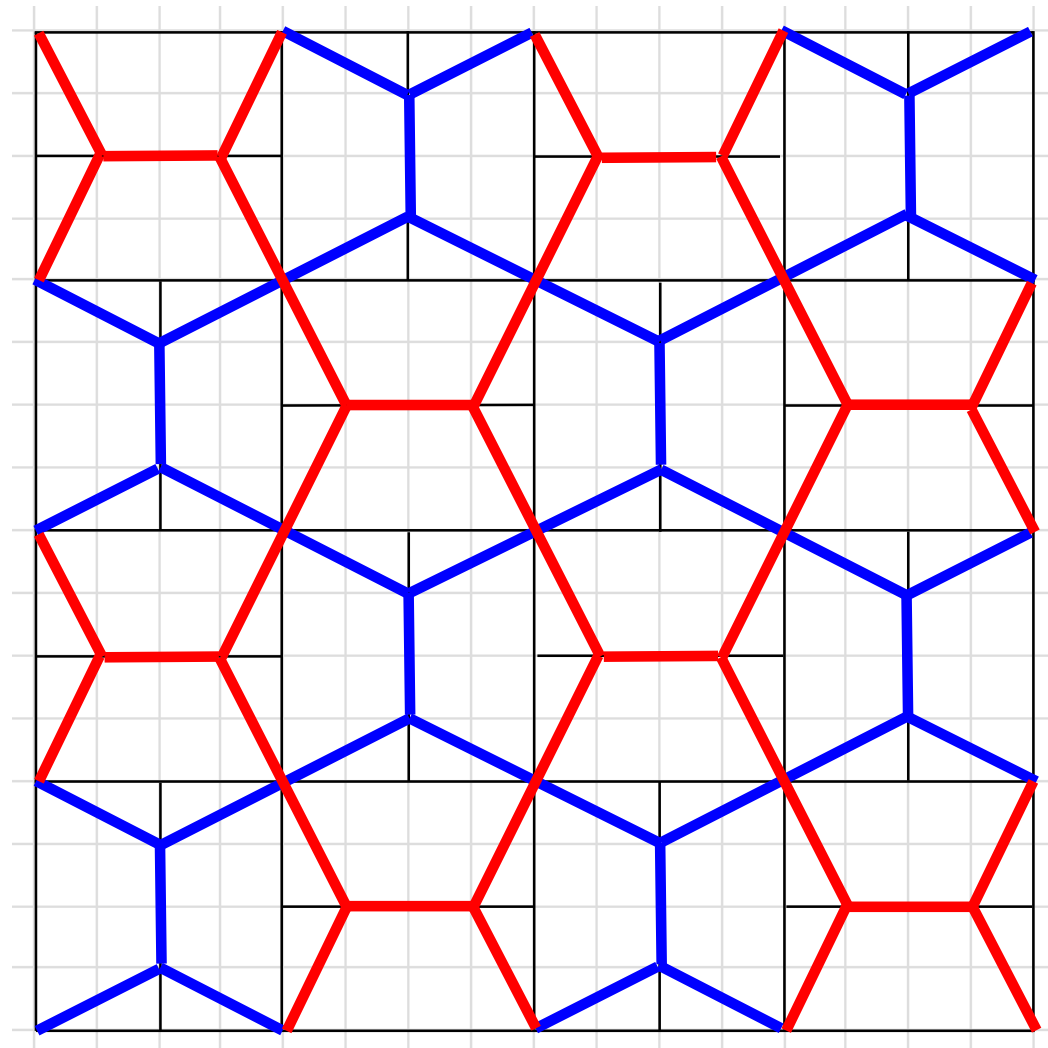
Cairo



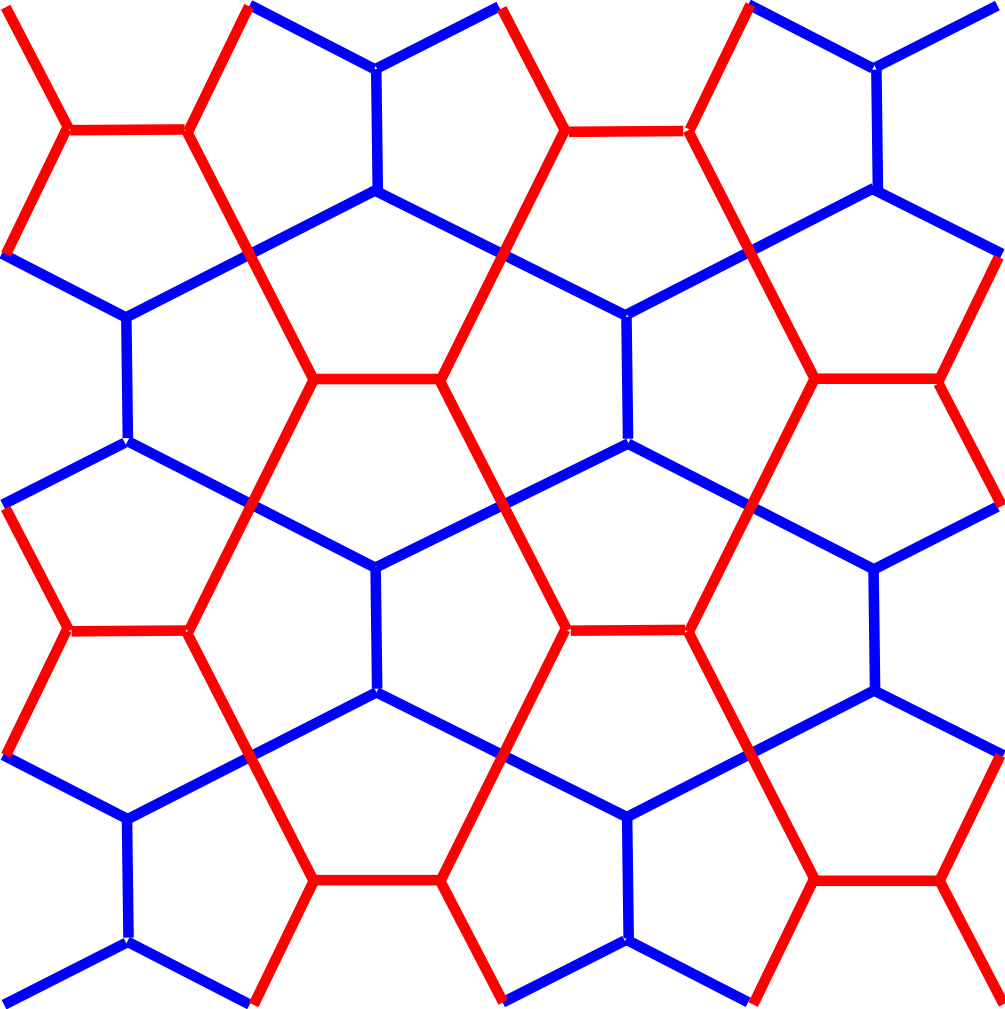
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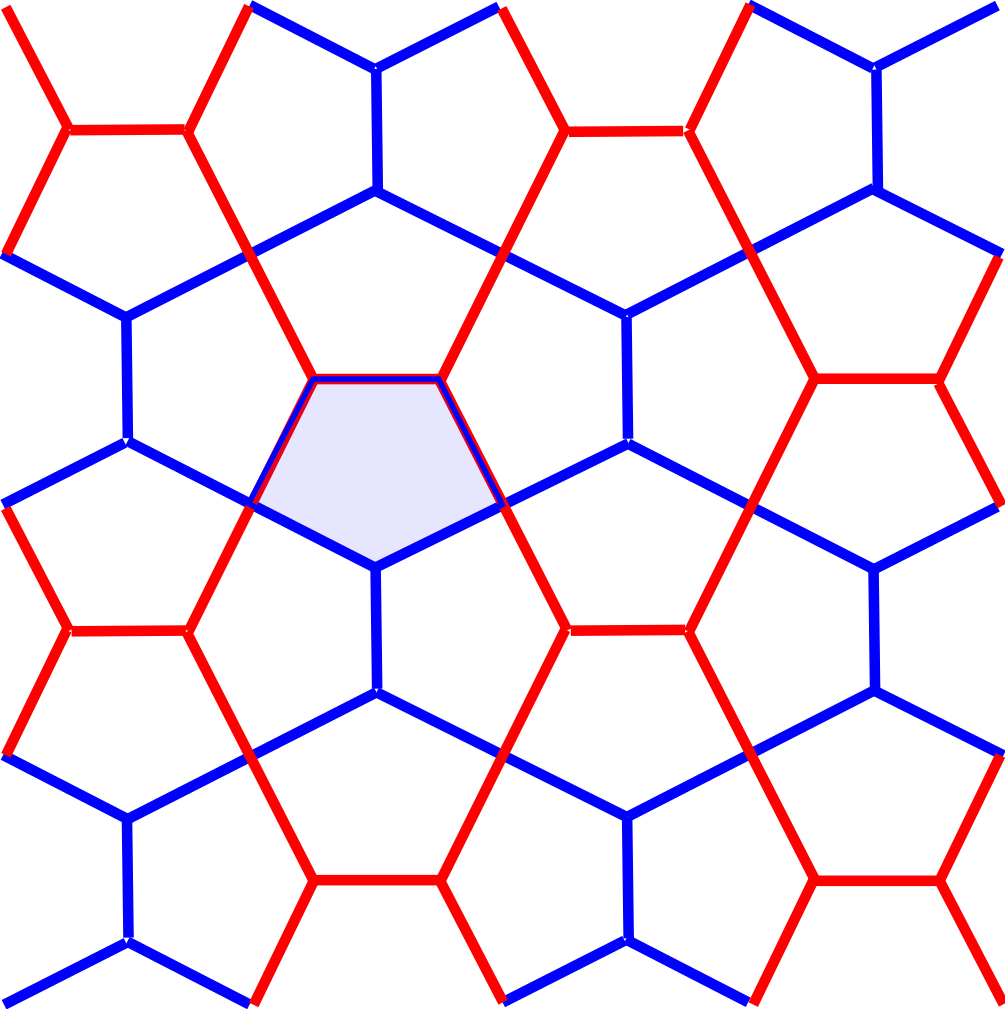
Cairo



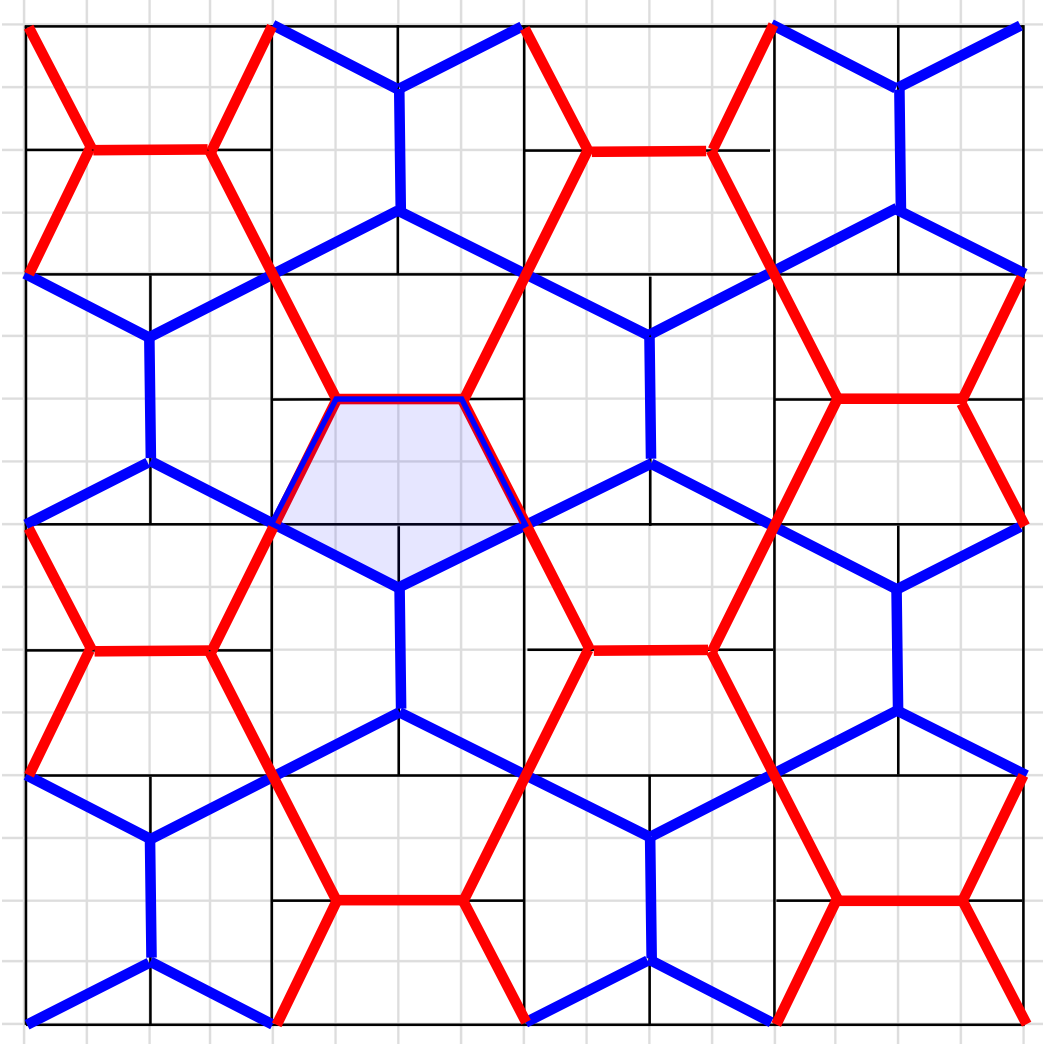
Cairo



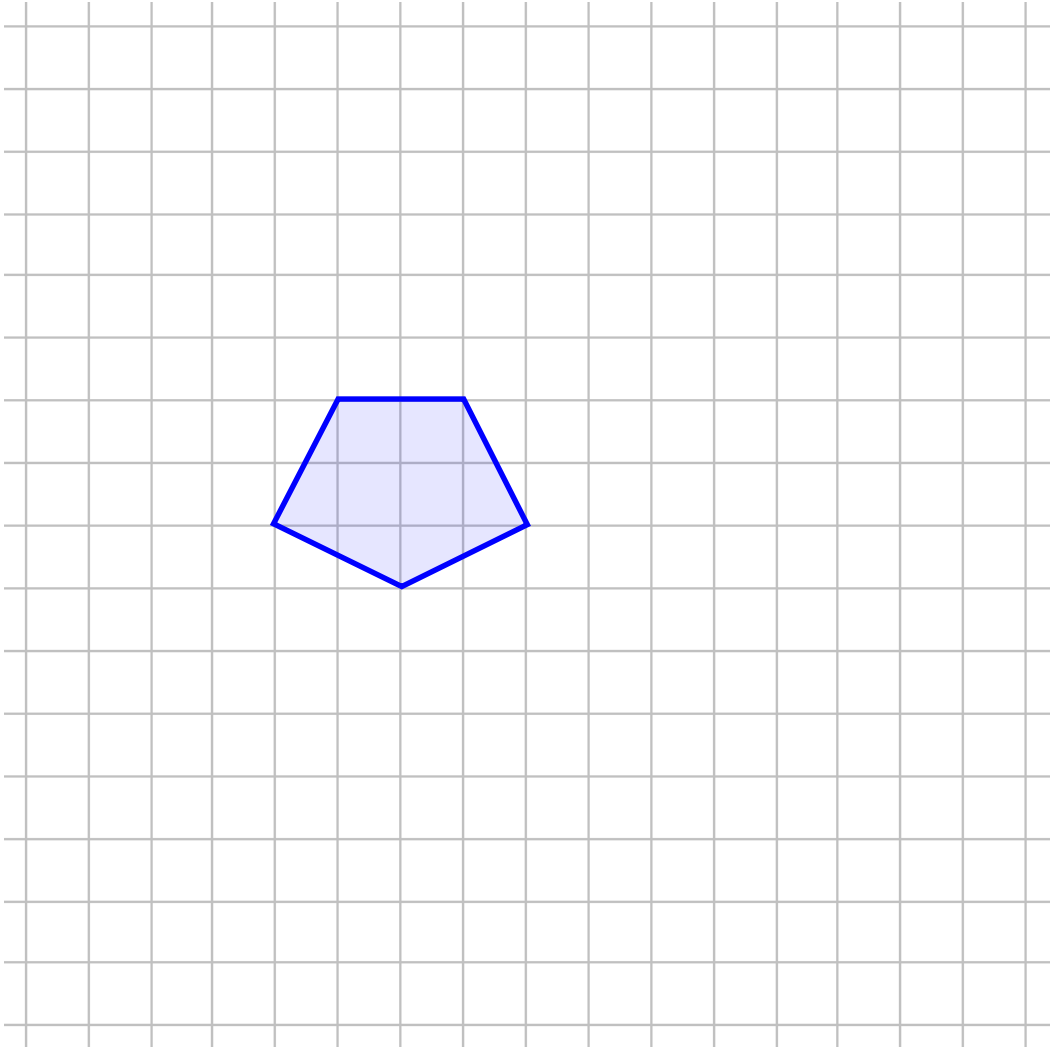
Cairo



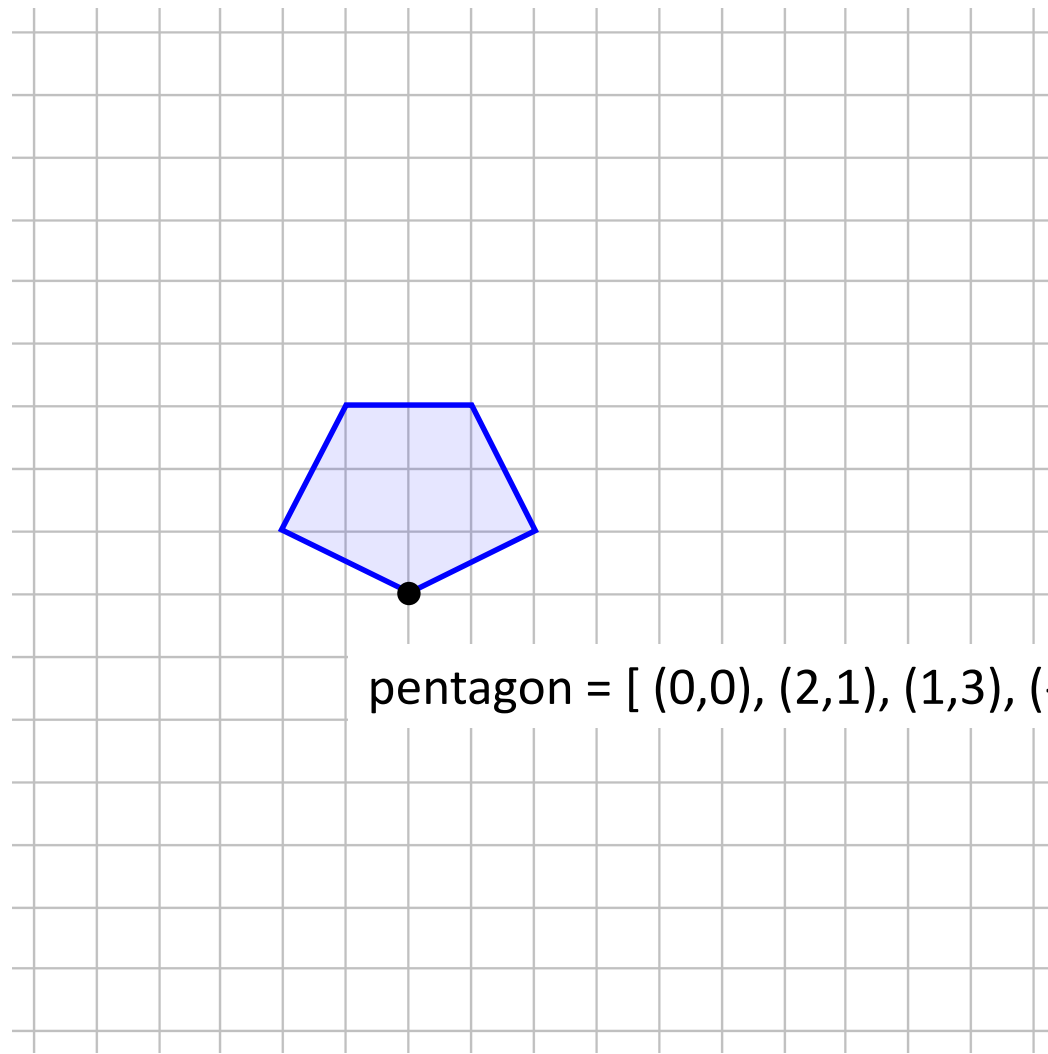
Cairo



Cairo

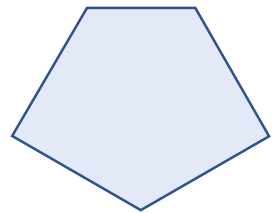


Cairo

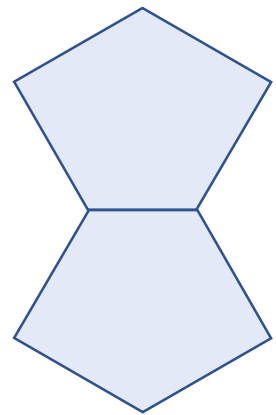


pentagon = [(0,0), (2,1), (1,3), (-1,3), (-2,1), (0,0)]

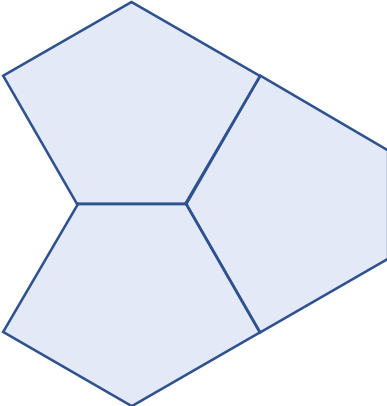
Cairo



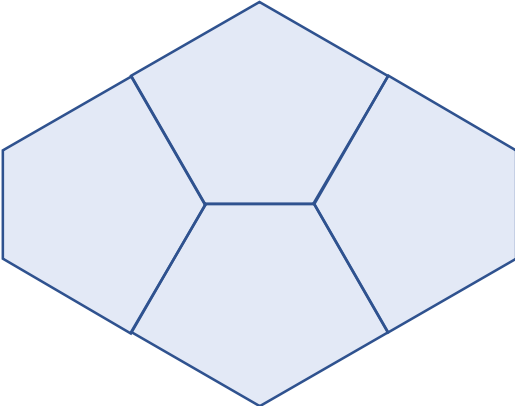
Cairo



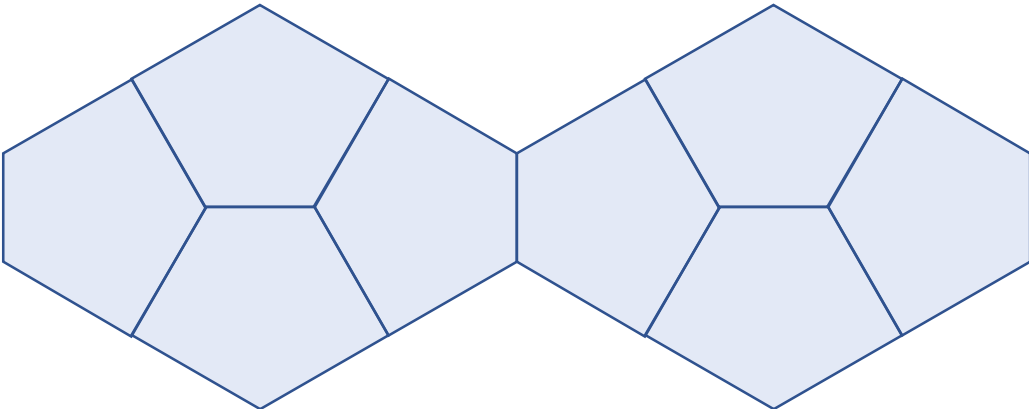
Cairo



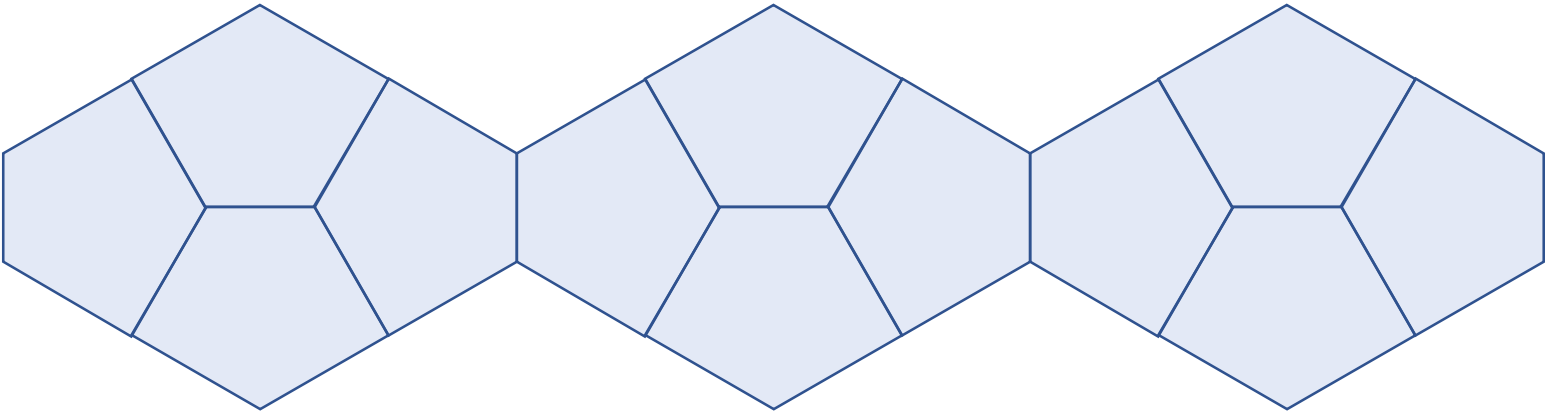
Cairo



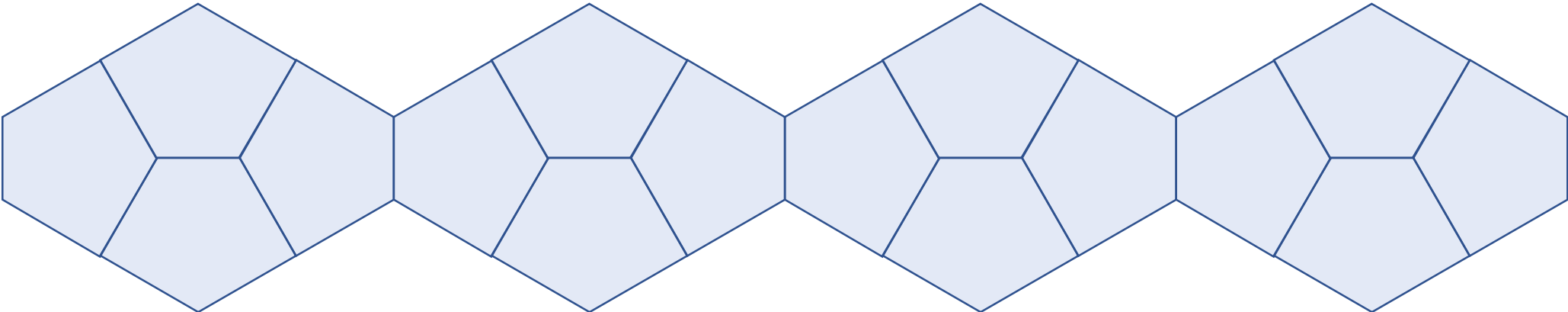
Cairo



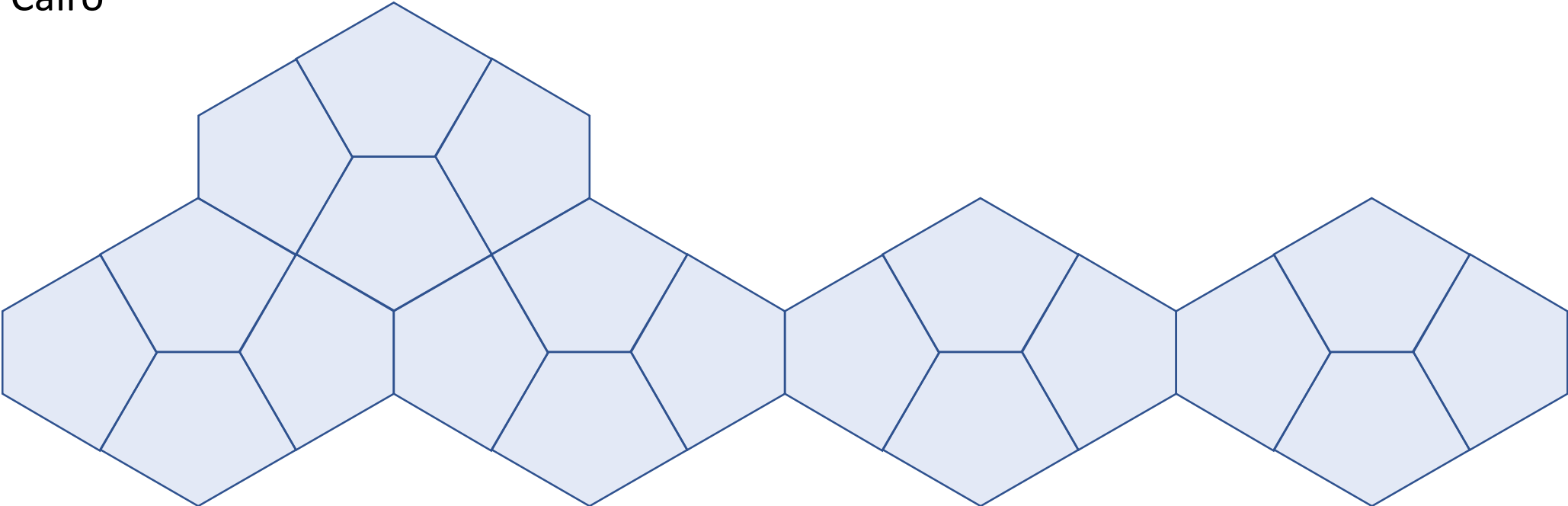
Cairo



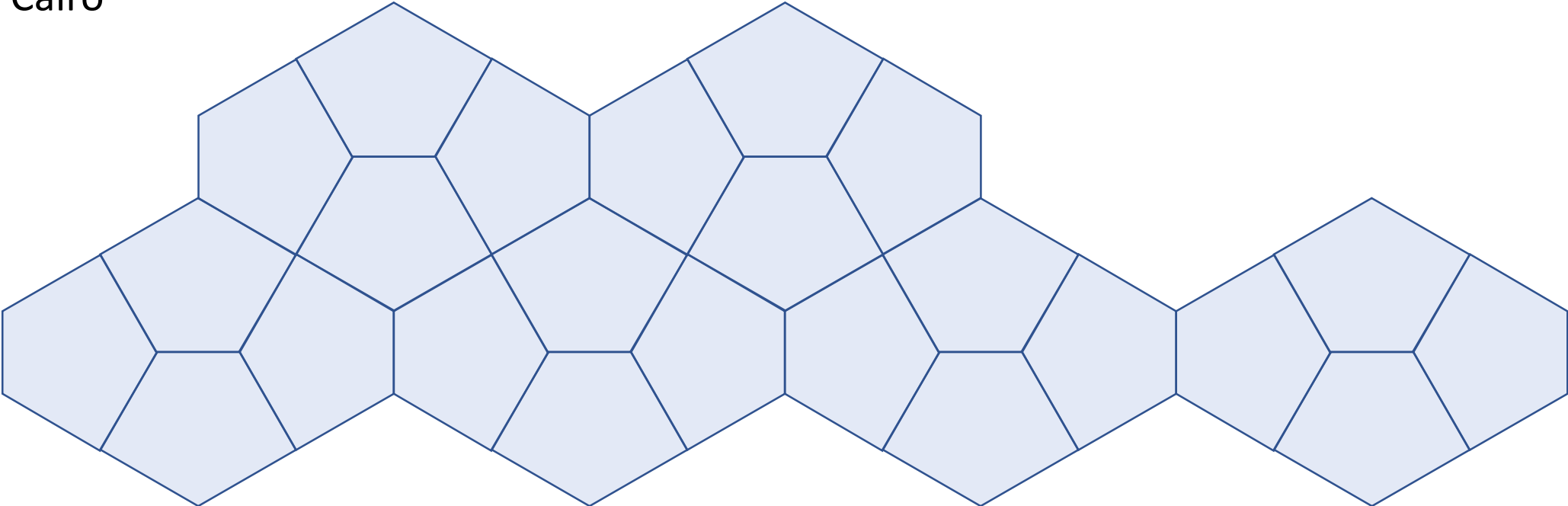
Cairo



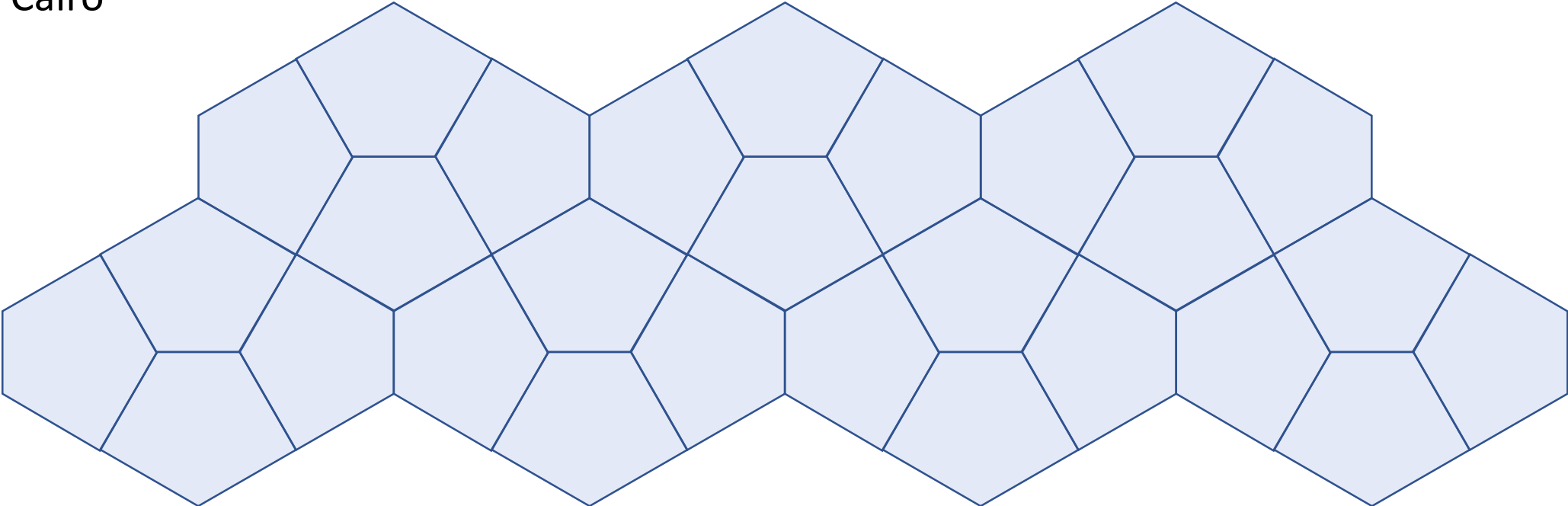
Cairo



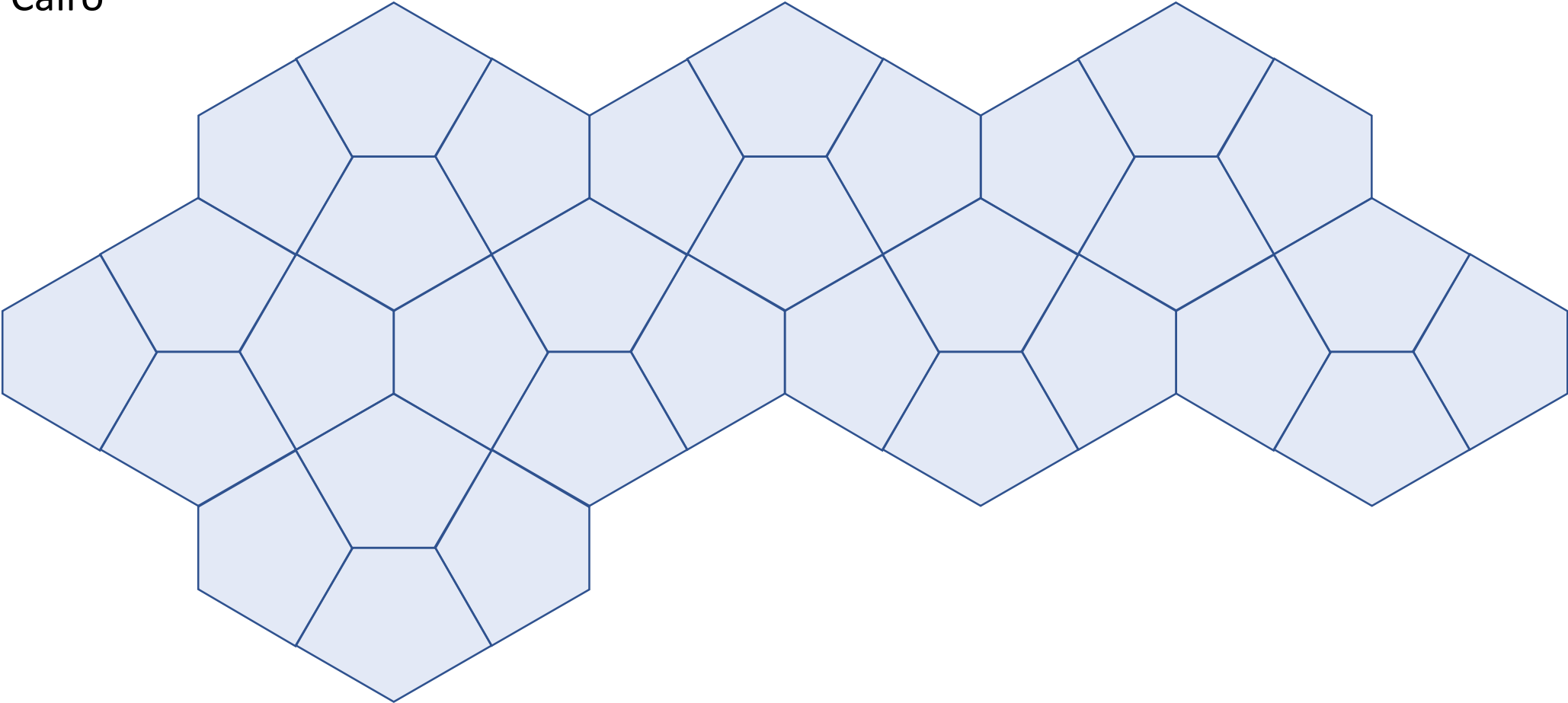
Cairo



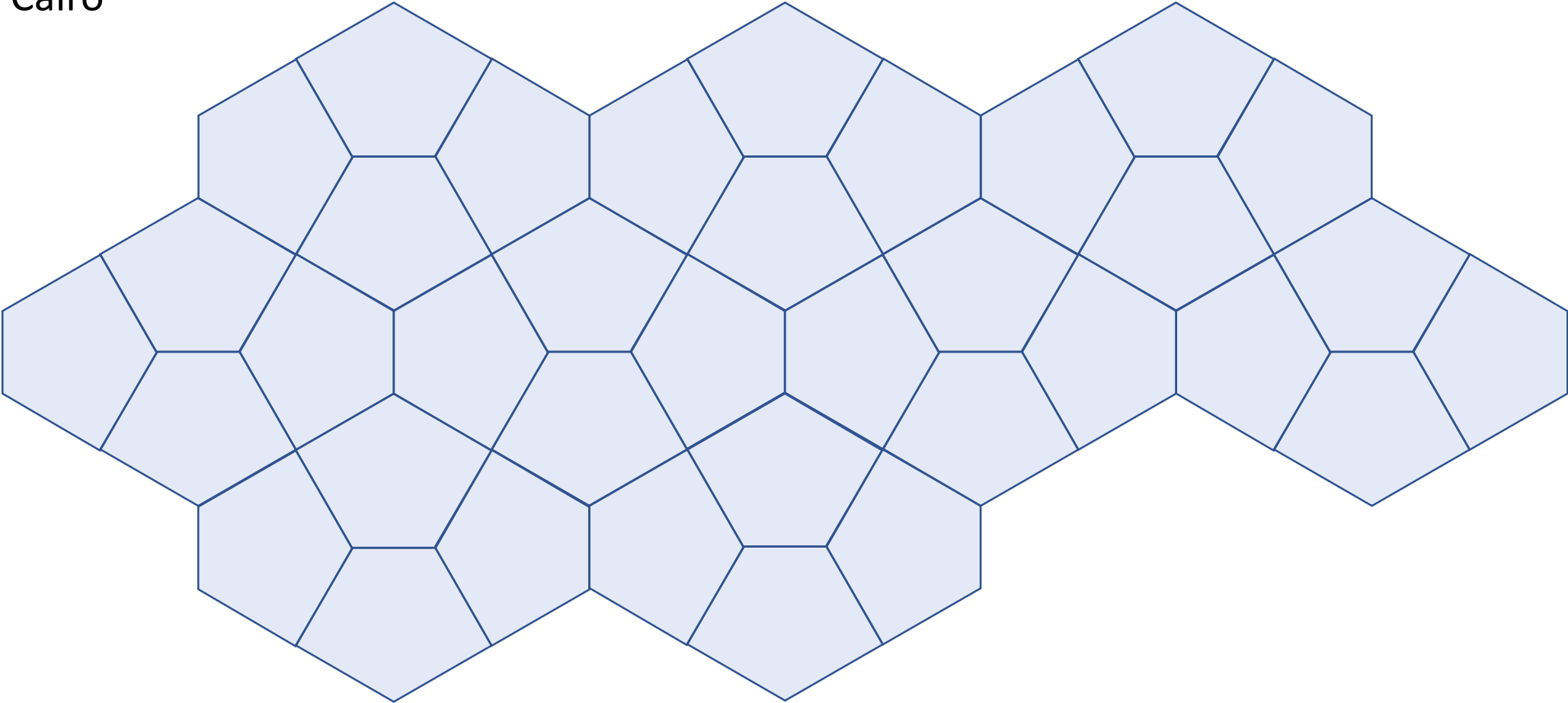
Cairo



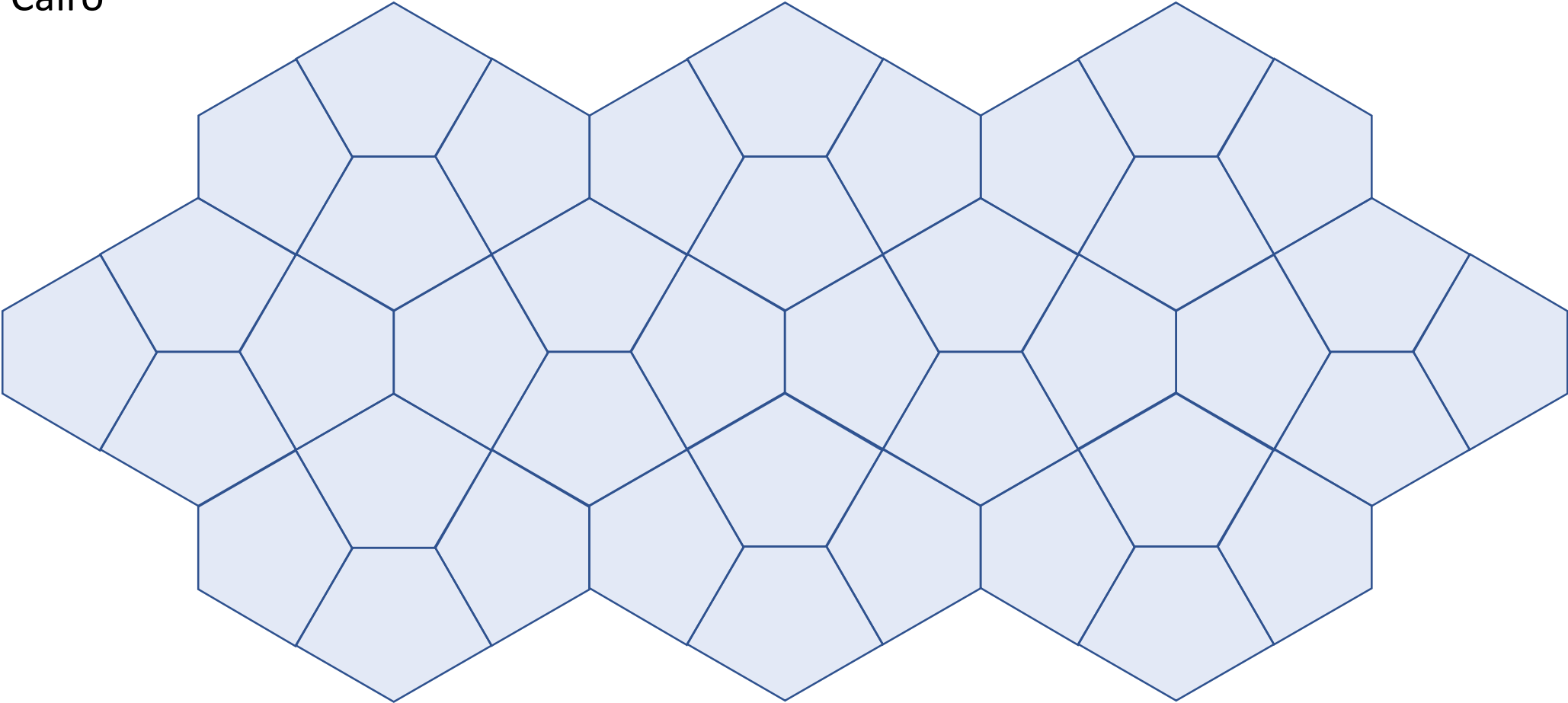
Cairo



Cairo



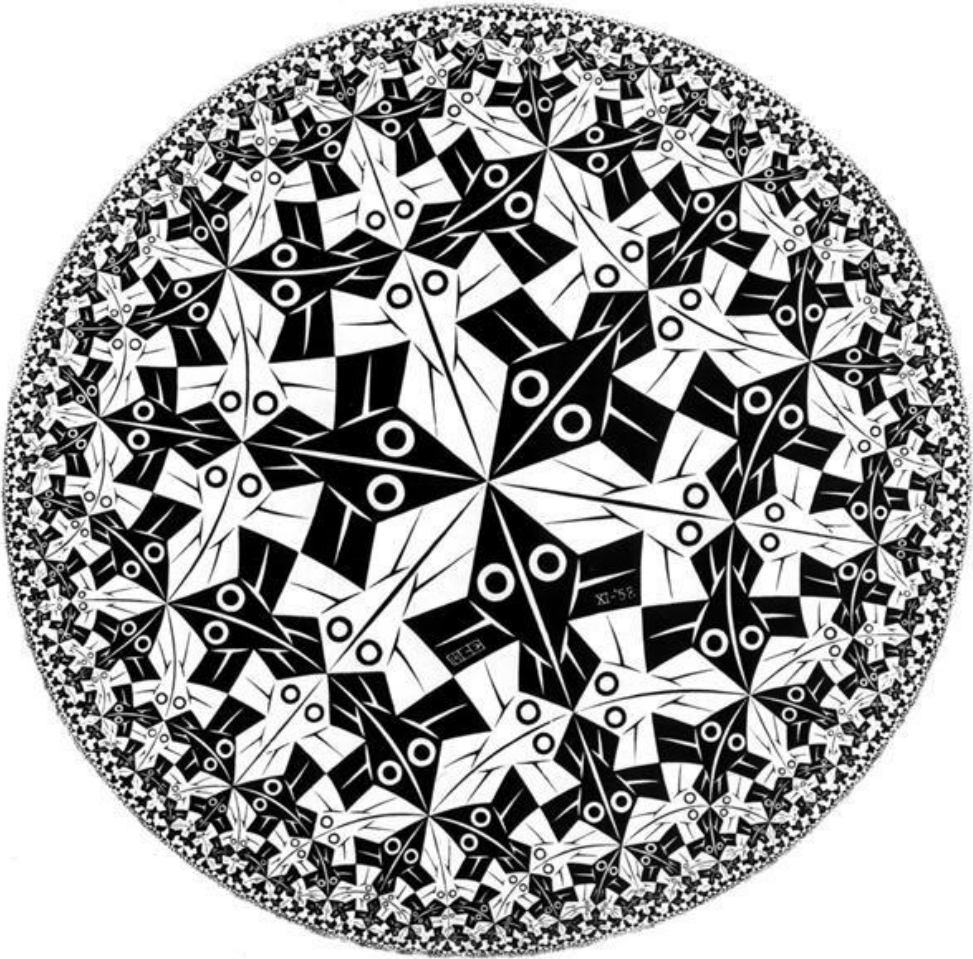
Cairo



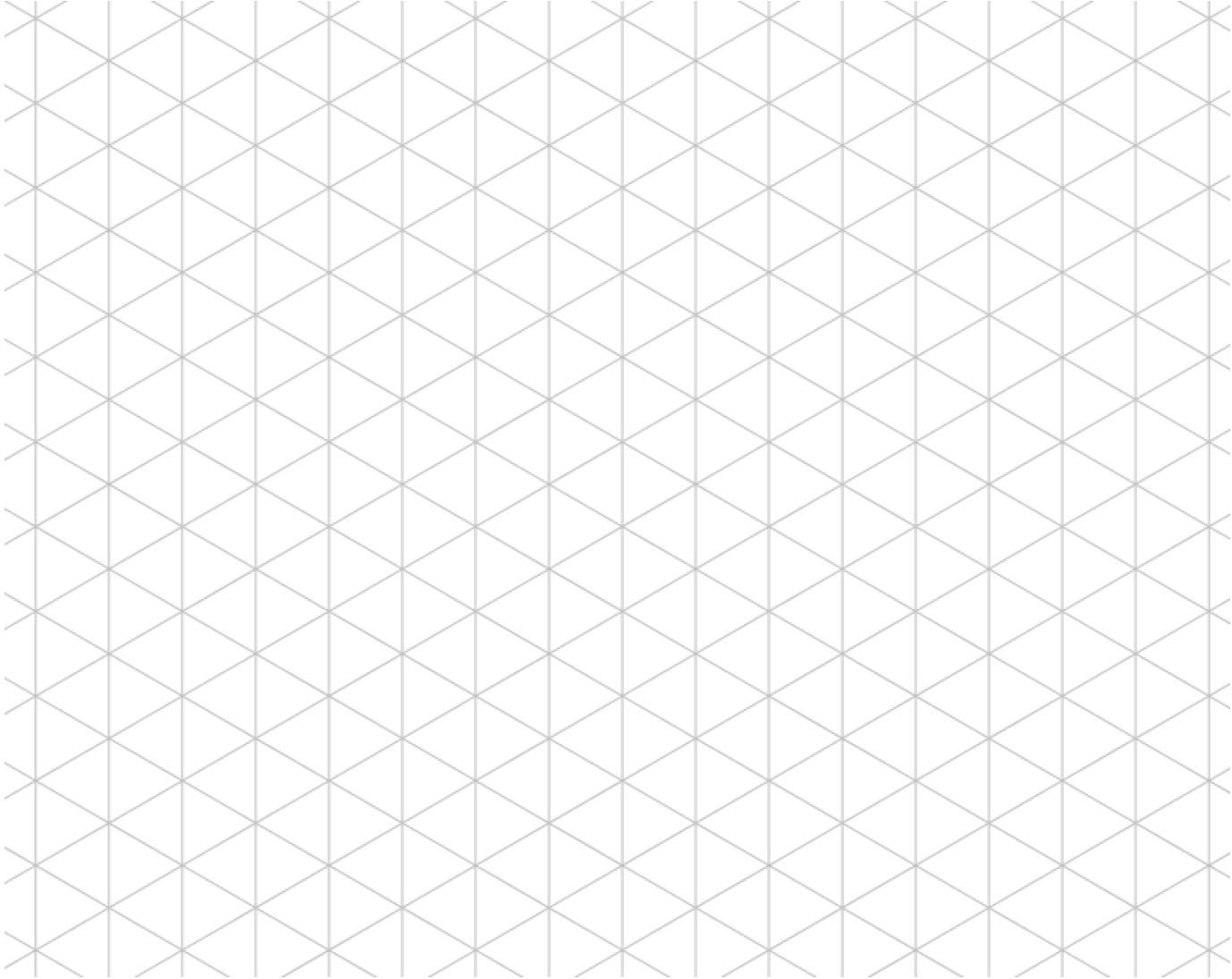
Cairo

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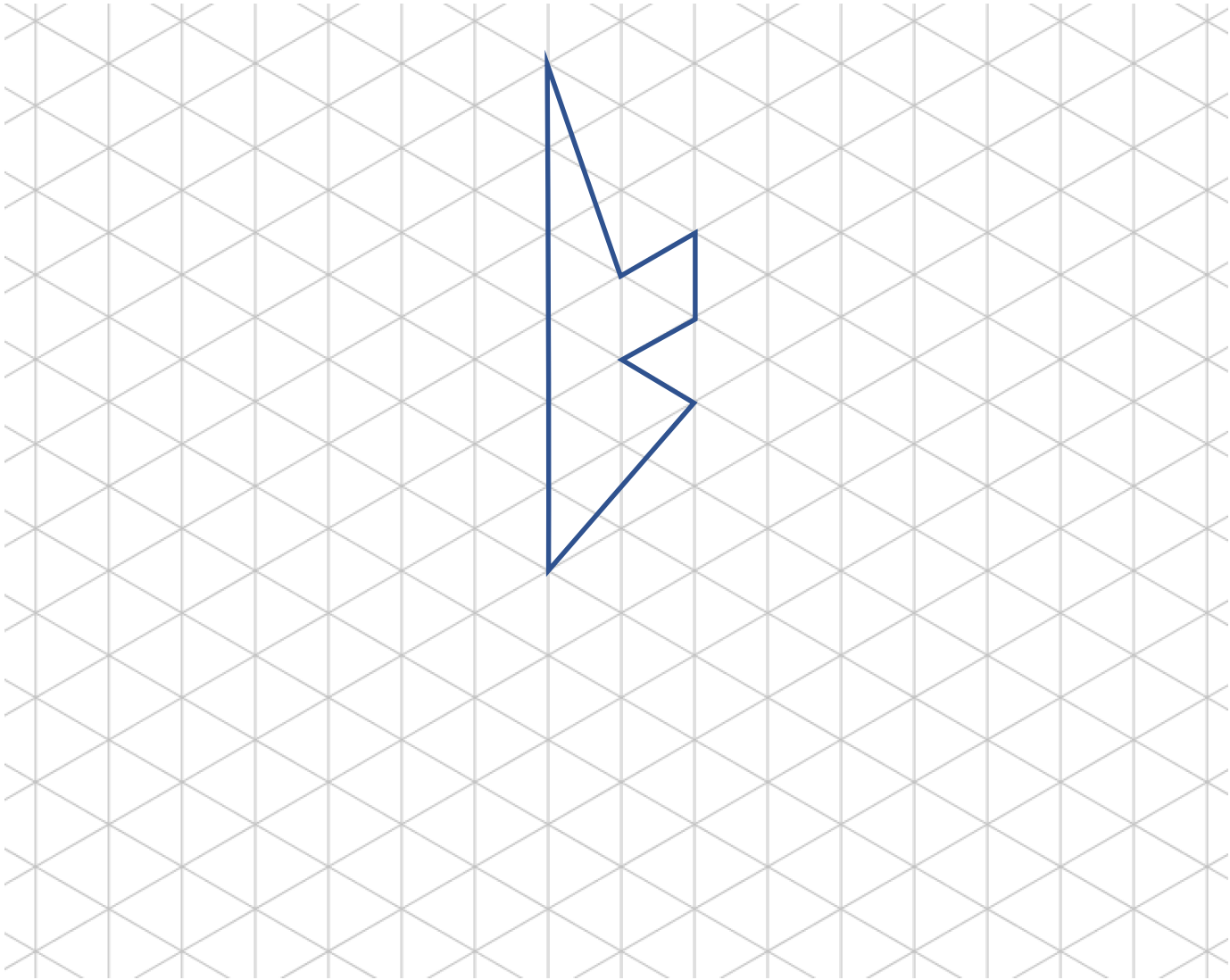
Fish



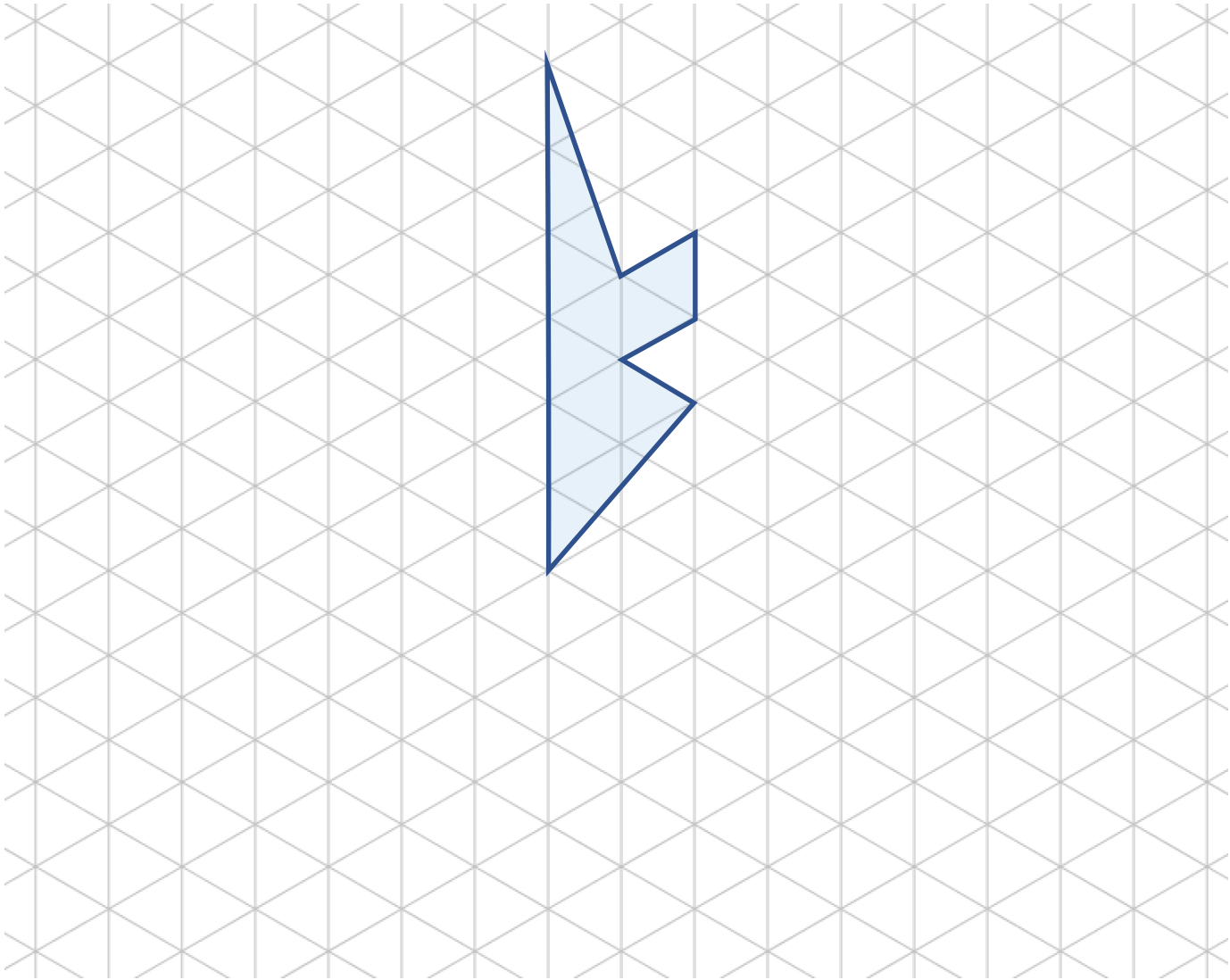
Fish



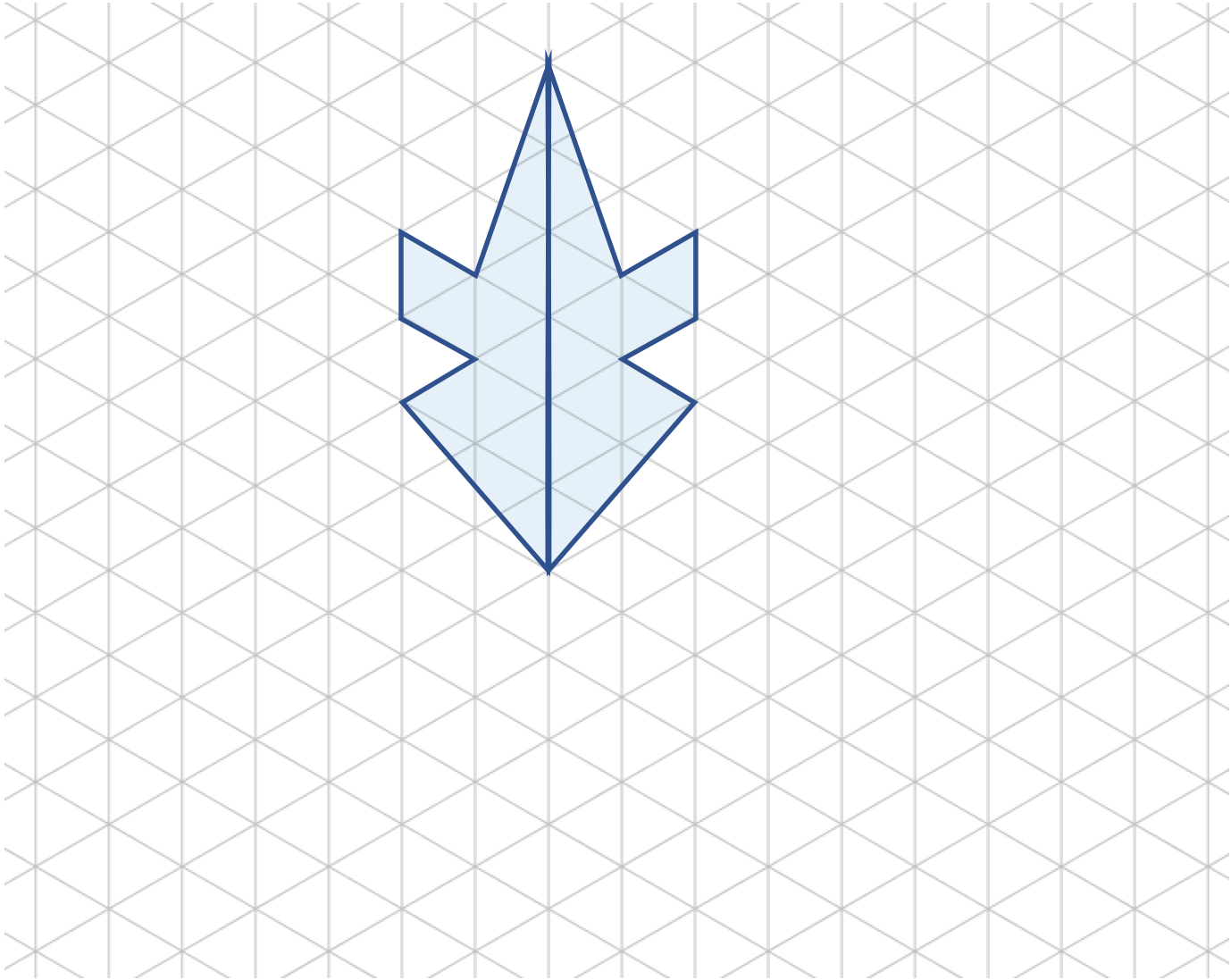
Fish



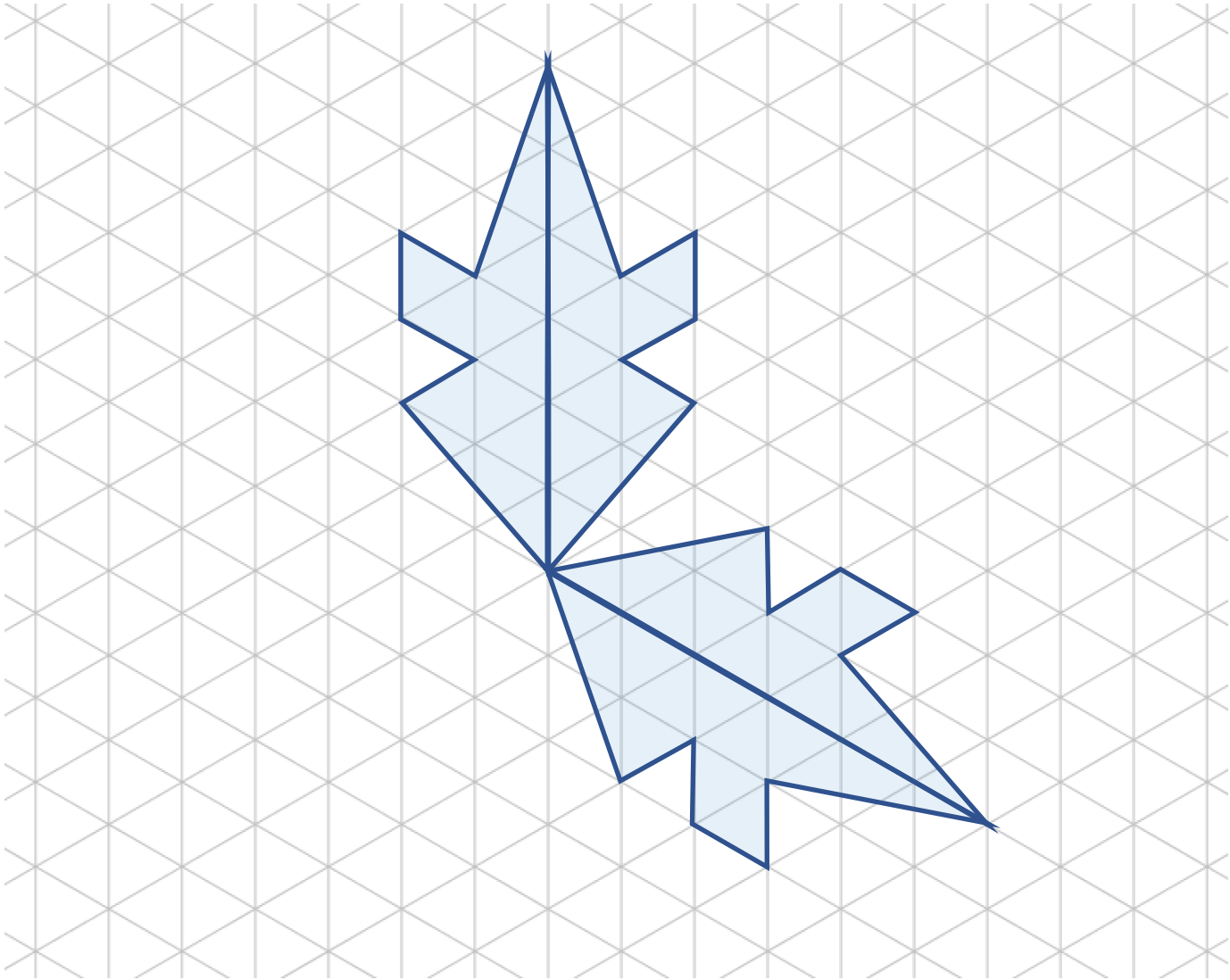
Fish



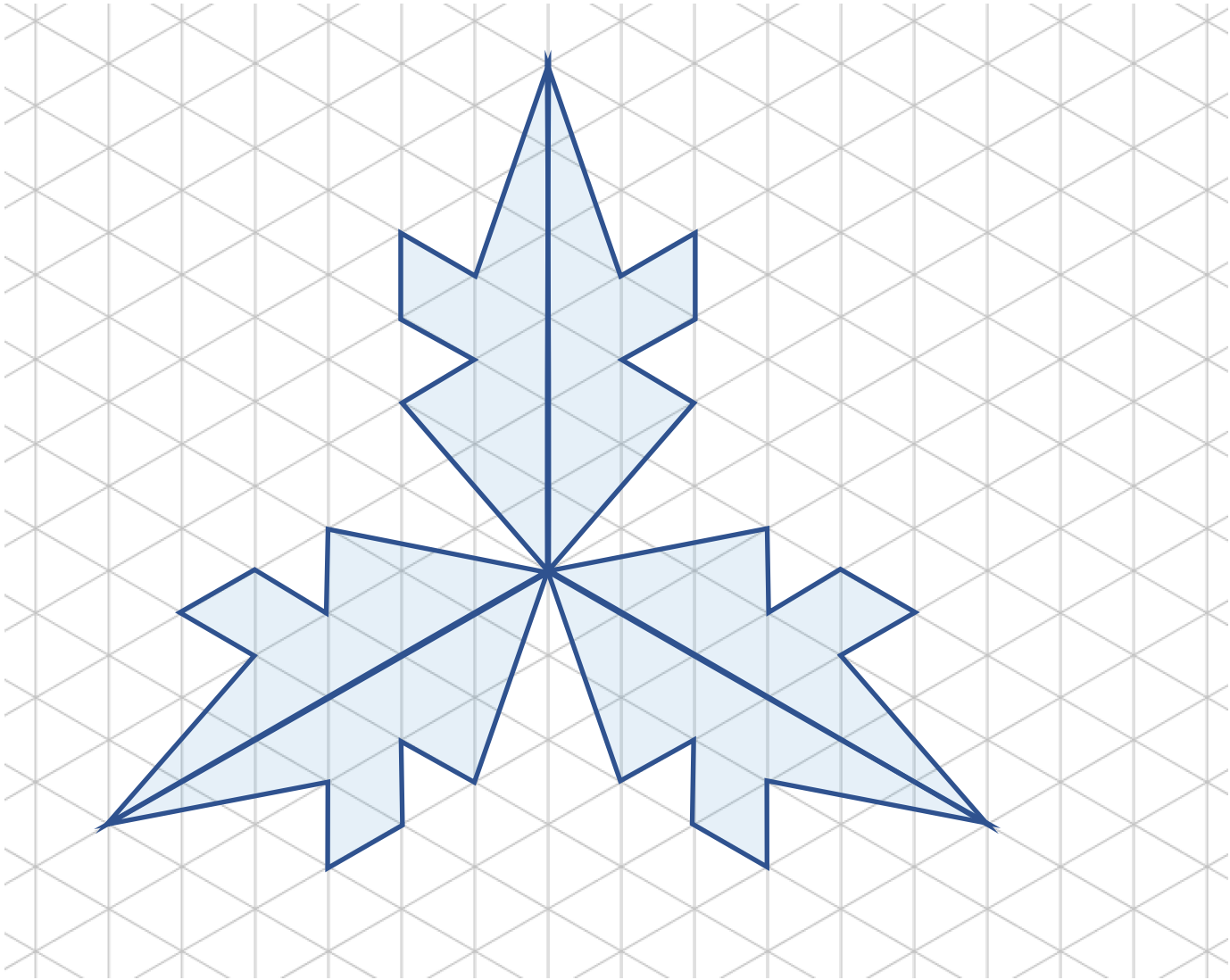
Fish



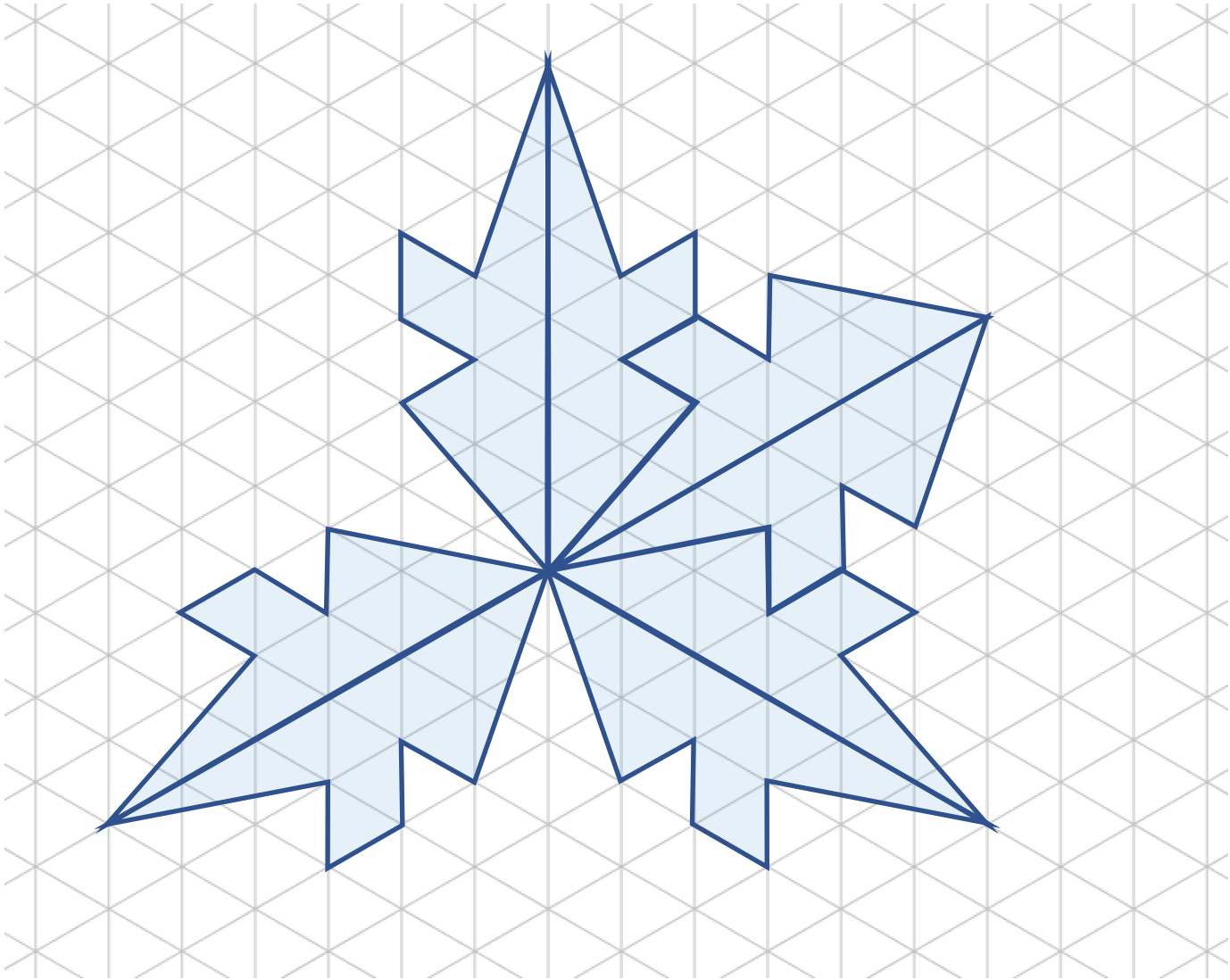
Fish



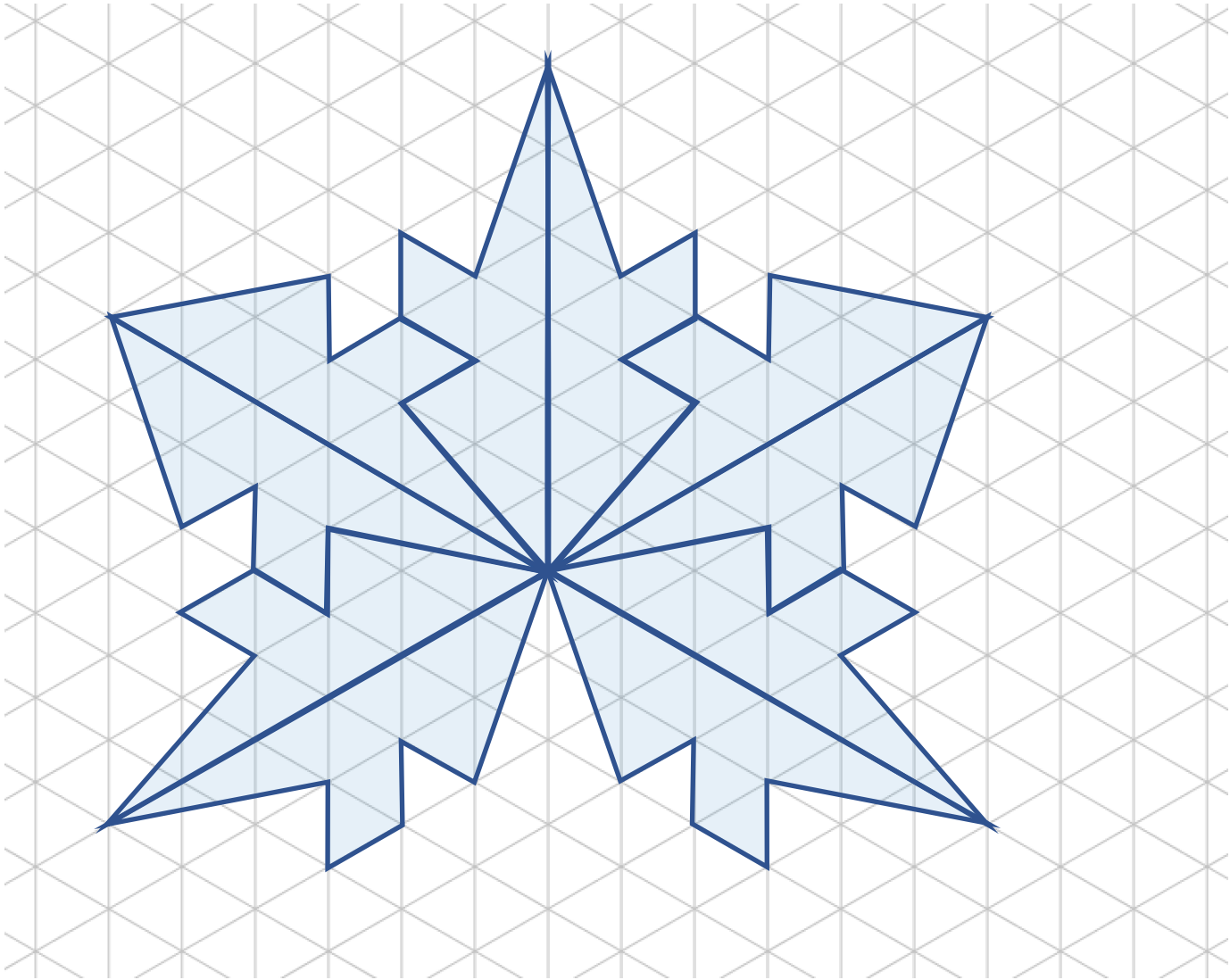
Fish



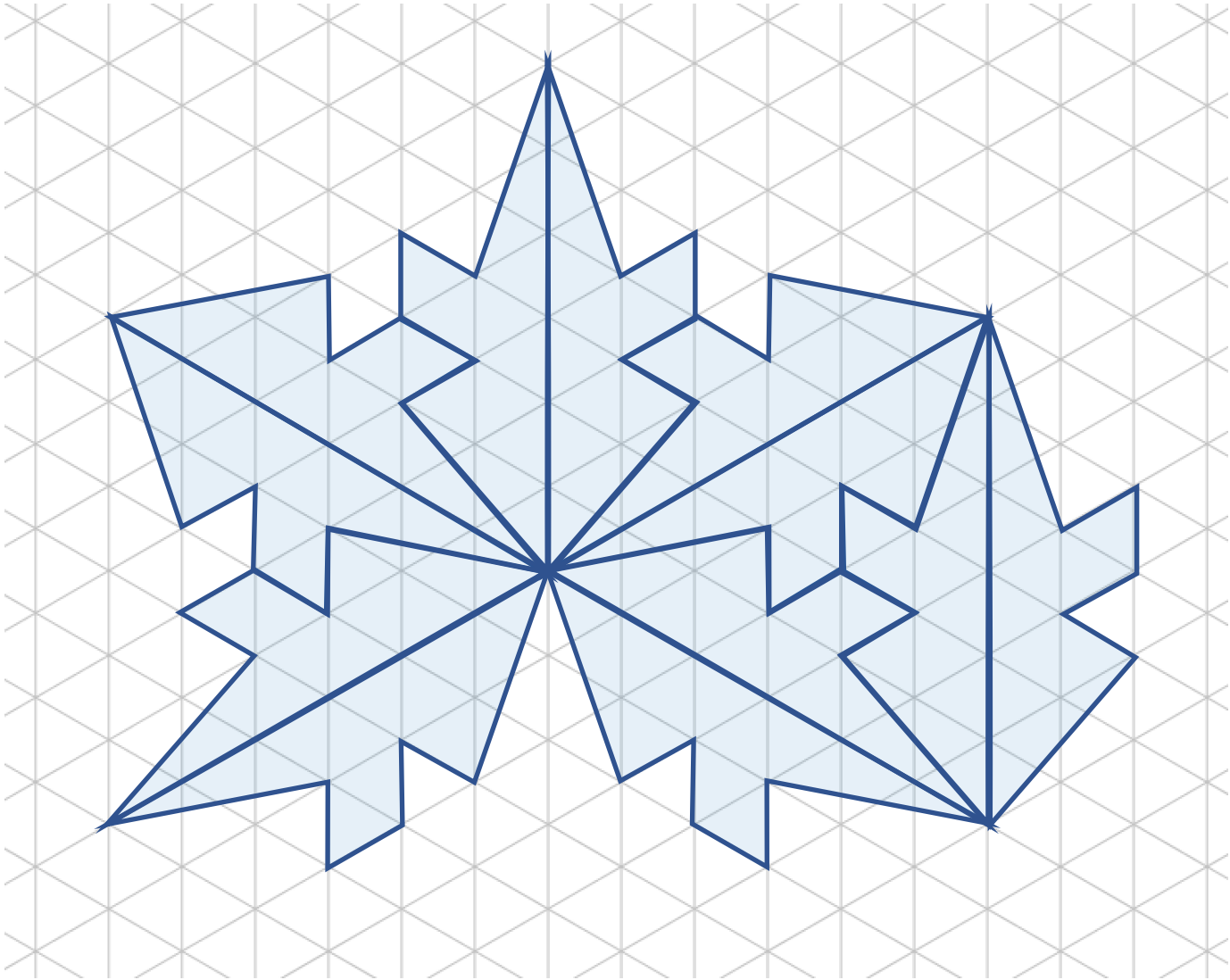
Fish



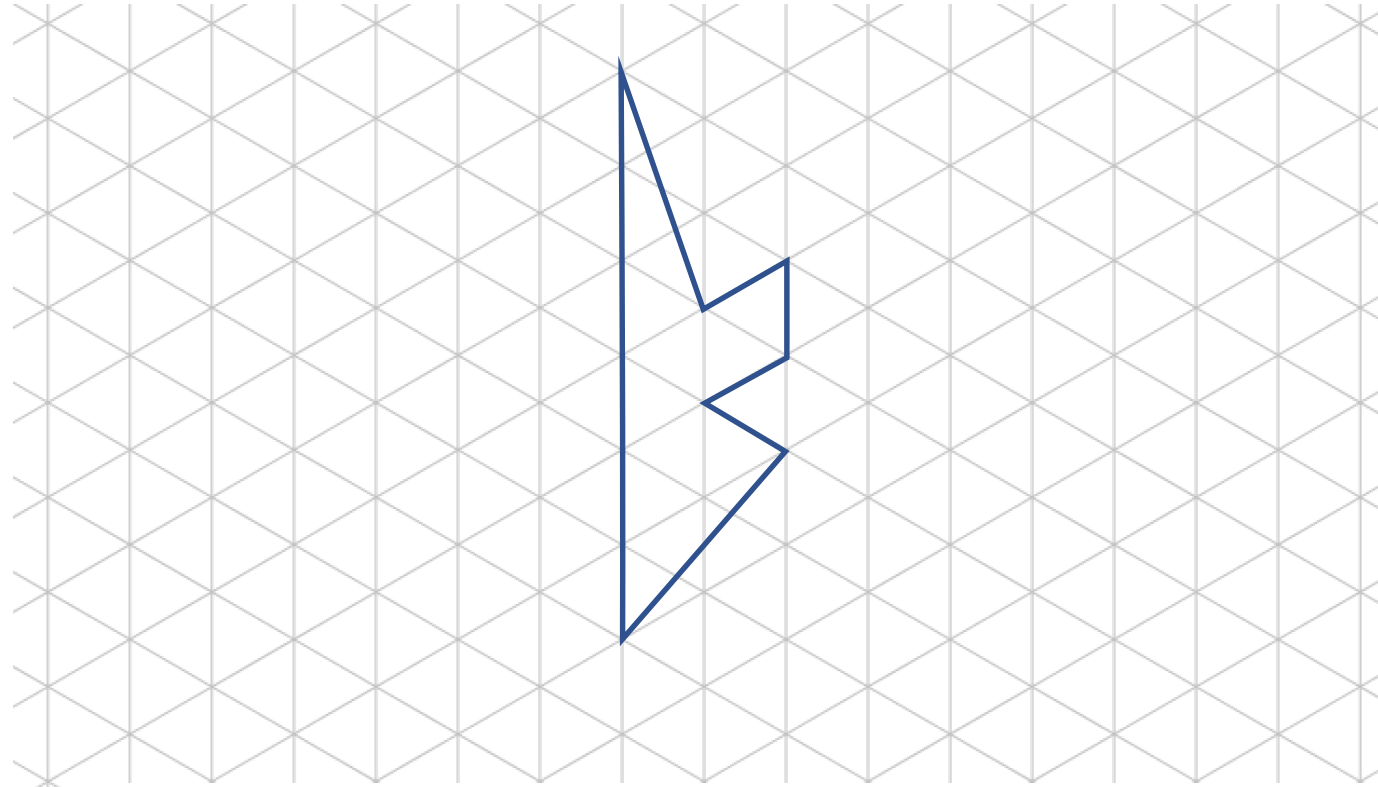
Fish



Fish



Fish



shape = [(0,0), (2,2), (1,2.5), (2,3), (2,4), 1,3.5), (0,6), (0,0)]

Fish

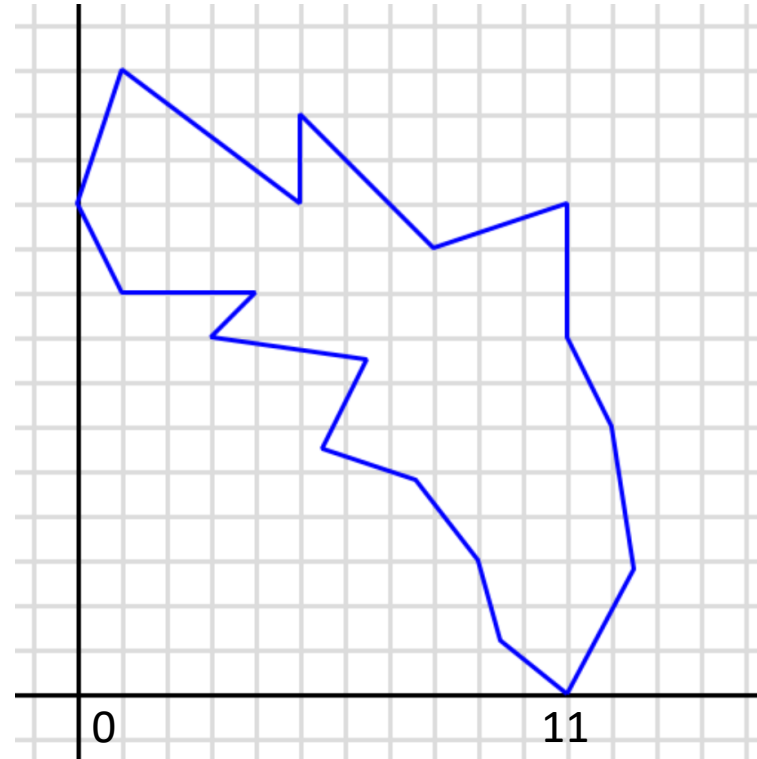
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Circular Fish



"Circular Fish", 1956 woodcut

Circular Fish

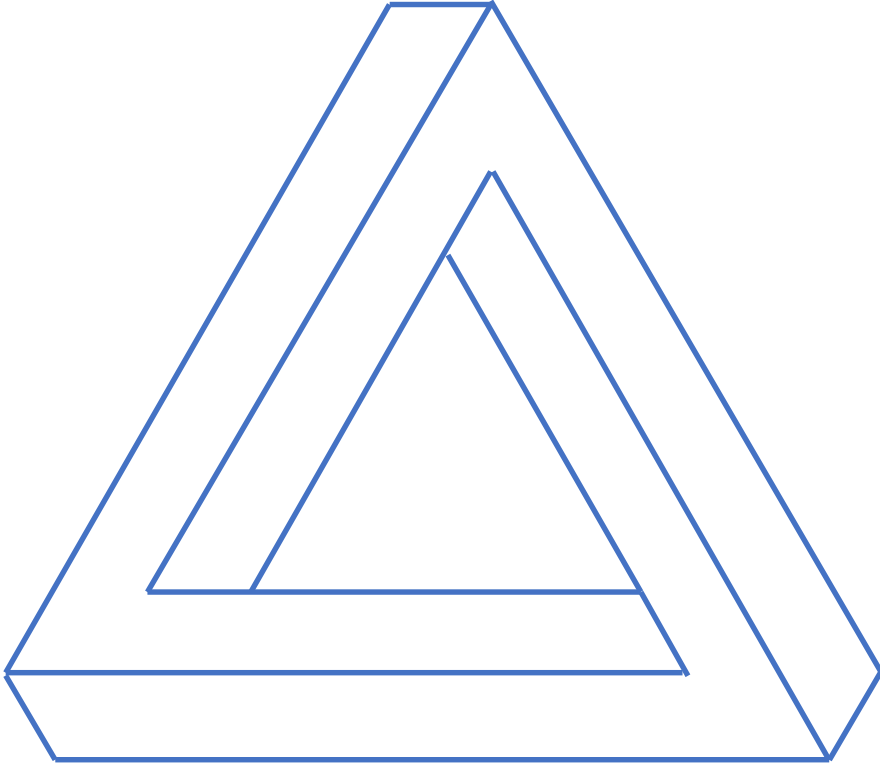


fish = [(11, 0), (12.5, 2.8), (12, 6), (11, 8), (11, 11), (8, 10), (5, 13), (5, 11), (1, 14), (0, 11), (1, 9), (4, 9),
(3, 8), (6.5, 7.5), (5.5, 5.5), (7.55, 4.8), (9, 3), (9.5, 1.2), (11, 0)]

Circular Fish

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Penrose



Penrose



Penrose



Penrose



Penrose



Penrose



Penrose

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Examples

Triangles

Birds



"Metamorphose II", 1940 woodcut