## Energy yield of a School PV-System

## 1. Introduction

## a. Learning Objectives

In the summer of 2011, a photovoltaic system was installed on our school property by the public utility company. The solar cell module was attached to a pole and could track the Sun. The energy generated is fed into the power grid via inverters. The data-output of the plant could be retrieved by colleagues on the Internet.

The interpretation of the diagrams as well as the evaluation of the data by working with a table is exemplified over the period of a day. The example is also suitable as a practical introduction to integral calculus.

## b. Scientific Background

The key physical point is the generation of electrical energy from solar energy, whereby the focus here is not on the processes in the solar cells, but on maximizing the energy yield. The solar module is designed in such a way that it can always be optimally aligned to the Sun's rays by rotating and tilting. Thus, one obtains an energy yield optimized by technology.

The values thus provided by the system are evaluated using mathematical methods. This way of dealing with tabular data forms the mathematical core of the exercise.
c. Connection with Sustainability Development Goals

This is mainly about Goal 7, clean and affordable energy.

## 2. The Background

In the summer of 2011, a photovoltaic system was erected on our school property. The solar cell module was attached to a pole and could track the Sun (rotated and tilted) (Fig. 1). It had an area of $51 \mathrm{~m}^{2}$ and a peak power of 7.35 kW . The system was set up in such a way that only in the winter months of December and January was there a slight shading from a part of the building. The energy generated was fed into the power grid via inverters for remuneration. However, this yield did not benefit the school, as the Bad Pyrmont public utility company had the system erected and also operated it. In the meantime, the plant has been moved again to allow for new construction in the school.

Since January 2012, the data of the plant could be accessed by teachers on the Internet, where they were presented very clearly (Fig. 2).


Figure 3 shows the time course of the electrical power for 2 weekdays (Friday, February 10, 2012, and Saturday, February 11, 2012). On Saturday, the sky was temporarily slightly cloudy; on Friday, however, the sun shone undisturbed the whole day. I shall now examine this day in more detail.

The system transmitted the current value of the power every half hour and created from it the diagram $\mathrm{P}(\mathrm{t})$, a polygonal curve, from which one could read the measured values (fig. 4 and table 1). From the bar chart in the upper right corner of Fig. 2 it can be seen that on Friday about 40 kWh of electrical energy E was fed into the power grid. The exact value is 39.29 kWh .

Fig. 4


| Zeit | 8.00 | 8.30 | 9.00 | 9.30 | 10.00 | 10.30 | 11.00 | 11.30 | 12.00 | 12.30 | 13.00 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| P/kW | 0,00 | 0,08 | 0,55 | 2,60 | 4,82 | 5,38 | 5,90 | 6,19 | 6,29 | 6,35 | 6,30 |
|  |  |  |  |  |  |  |  |  |  |  |  |
| Zeit | 13.30 | 14.00 | 14.30 | 15.00 | 15.30 | 16.00 | 16.30 | 17.00 | 17.30 | 18.00 |  |
| P/kW | 6,23 | 6,03 | 5,91 | 5,42 | 4,81 | 4,33 | 1,34 | 0,32 | 0,08 | 0,00 |  |

Table 1

## 3. Exercises and Questions

a. Plot the table data graphically as a connected scatter plot on the calculator (horizontal axis: time $t$, vertical axis: power $P$ ).

How can you explain the shape of the curve?
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$\qquad$
Why does the curve have a small indentation on the right?
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$\qquad$
b. For the power P during a period $\Delta \mathrm{t}$ : $\boldsymbol{P}=\frac{\Delta E}{\Delta t}$.

How do you get the energy $\Delta \mathrm{E}$ from the power?
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$\qquad$

How do you get the total energy E generated in a day?
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$\qquad$
c. The energy is now to be illustrated graphically. For this purpose, only the interval from 9:30 to 10:00 is to be considered at first.

Consider how the electrical energy generated for this period would have to be represented graphically in the diagram (sketch or text)!
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$\qquad$
$\qquad$
Justify why you should not use only the left or the right end of the interval in this representation!
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$\qquad$
d. Now the entire diagram is to be considered (see picture). The generated electric energy is represented by the area below the red polygon. This area can be calculated approximately - by the lower sum (dark blue) or by the upper sum (light and dark blue). For this purpose, two additional columns in the table must be filled in.

Think about which values belong in these columns,
 so that you can show lower and upper sum in the graph!

Now calculate the energy yield of one day for upper and lower sum! Note that the measurements are made every half hour! (To simplify the calculation you can use the command cumsum()).

Why is the actual value of 39.29 kWh between these values?
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$\qquad$

Think about an improvement and implement it!
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(Hint: A better approximation can be expected if the area under the polygon course is calculated more precisely. It is composed of trapezoids, which can be easily calculated with the data already generated by taking the average of e3 and e4 in another column e5. This gives the approximation $\mathrm{E}=39.465 \mathrm{kWh}$. Since the deviation from the actual value is only about $0.5 \%$, this is a very good approximation, which cannot be further improved with the available data material. The small deviation can probably be explained by the fact that in reality $\Delta t$ should be much smaller than 30 minutes).

