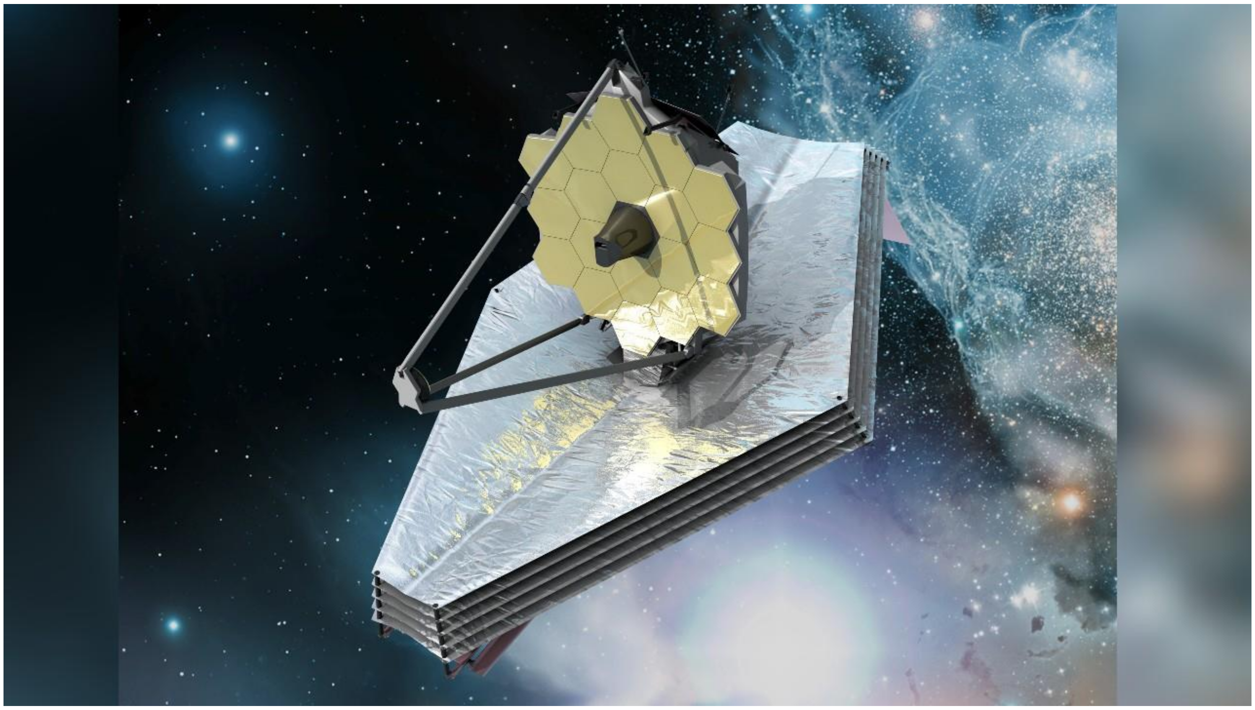
The James Webb Telescope and Lagrange Points



The James Webb Telescope, JWT, is the most powerful and most complex telescope ever launched into space. Its infra-red cameras will be able to observe galaxies which formed shortly after the Big Bang and will need to be protected from the heat of the Sun. This is why it has been sent to Lagrange point 2, 1.5 million kilometres from the Earth and well beyond the orbit of the Moon. But what are these Lagrange points and why do they help telescopes like the JWT and the SOHO observatory which is looking at the Sun?

Out in space it is important to know the strength of the gravitational field around you as it is this which will help guide your motion. At Lagrange points 1 and 2 the gravitational field of Earth and Sun combined assure that a satellite placed there will have an orbital period of one year. In principle an object placed there will orbit forever at this fixed displacement from the Earth. In practice however objects will slide away from it and inevitably end up either in the Sun or on the Earth. It is like trying to balance something on the end of a pencil. Small rocket thrusters are required to keep JWT in place and in fact it will be sent into a special orbit (a halo orbit) around the L2 point to keep fuel consumption to a minimum.

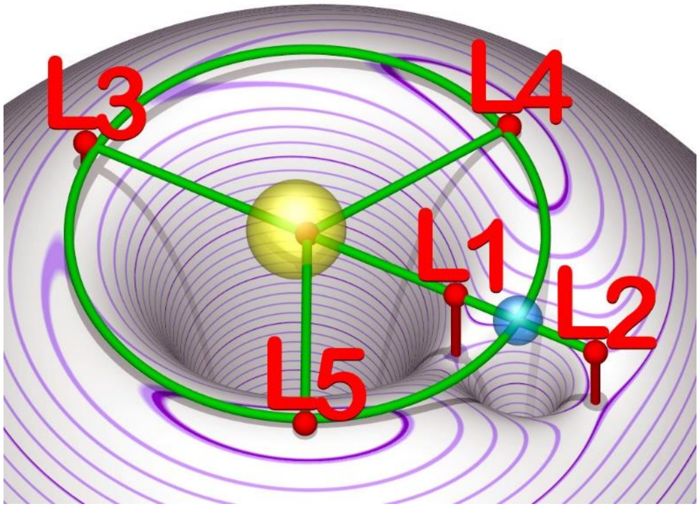


Figure 1 from wikipedia Lagrange points and equipotentials

Figure 1 shows the Sun and Earth with equipotentials drawn showing clearly the saddle points at L1,2 and 3.

Lagrange points

In a two body system such as the Sun And Earth there are 5 special points where a small object will orbit the Sun with the same orbital period as the Earth and so maintain a fixed position in space relative to the Earth. These are the Lagrange points.

Euler (Swiss mathematician, 1707-1783) discovered the first three points but then **Lagrange (Italian mathematician, naturalized French, 1736-1813)** discovered all 5 of these special points by plotting out the gravitational energy contours of a two body system.

L4 and L5 are hills while M1 and M2 sit at the bottom of gravitational wells. L1 L2 and L3 are saddle points in this two dimensional plot.

Figure 2 shows the contours where M1 is much larger than M2.

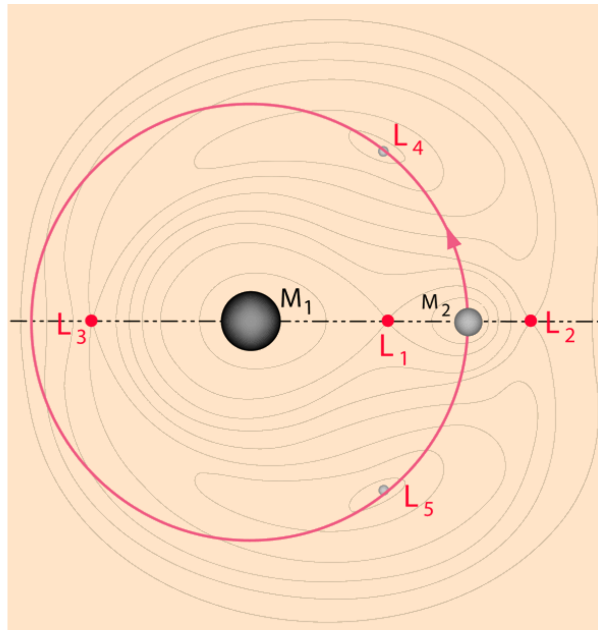


Figure 2 from Hyper Physics

Curiously because the system is rotating around M1, L4 and L5 turn out to be stable, in that an object at those points if nudged slightly will orbit (so called halo orbit) around the points. L1 L2 and L3 being points on saddle shaped regions are unstable in that an object at those points if nudged slightly will end up either in the Sun or on the Earth! However L1 and L2 are useful for placing satellites which need to maintain a fixed distant position relative to Earth. (Geostationary satellites sit inside the Earth's potential well at a point where the orbital period around the Earth is 24 hours, or one day.)

Lagrange 1 and 2

It is fairly easy to understand how L1 and L2 arise. Normally an object orbiting the Sun closer than the Earth will orbit at a higher velocity than the Earth because of the increased gravitational force. At L1 the pull of the Earth weakens the pull of the Sun and so the object at L1 will move at a lower velocity with a period equal to one year.

Similarly, beyond the Earth an object will orbit more slowly being in a weaker gravitational field. At L2 however the pull of the Earth adds to the pull of the Sun so increasing the gravitational field enabling an object to move faster and so orbit with a one year period.

Calculating the positions of these points is more difficult but the tns file, Lagrange, will help you to find L1 and L2 using nothing more than high school physics and TInspire.

An object at L2 is in the Earth's shadow and so protected from the fierce radiation of the Sun. This is the position of choice for the JWT as it will operate in the infra-red region of the spectrum, meaning that elevated temperatures will affect the imaging systems.

An object at L1 is closer to the Sun and so will detect damaging solar storms before they can reach Earth. This is the position of choice for the solar observatory SOHO. Electricity networks can be shut down quickly if SOHO signals an electrical storm is on the way. In 1989 Quebec suffered a nine hour blackout because of a solar storm.

Lagrange tns file.

The model is an inverse square with one 'mass' 100 times the other. Most accounts explaining the origins of the Lagrange points confuse gravitational potential with gravitational force. They also confuse Inertial reference frames with rotating reference frames and so talk about Coriolis forces and centrifugal forces as though they are real forces. School physics syllabuses usually insist that orbital motion is caused by a centripetal force, the force due to gravity and so use inertial reference frames. Force will be used rather than energy to find the position of the two points.

The approach is to consider the way in which the Earth's gravitational field influences the Sun's field by simply summing the two fields. Only the field on a straight line from the Sun through the Earth is considered. The Earth's field has been drawn as negative on the Sun side and positive on the other side because of course all gravitational forces are attractive. This means that the Sun's field is decreased between Earth and Sun and increased beyond the Earth.

A moveable point on the resultant or net field, screen 13 will provide the strength of the field (g) experience by a small satellite at that position (x). The equations are hidden but can be found in the history. The trick is to find the position x where the orbital period for the satellite is the same as the orbital period of the Earth. This is achieved by comparing the centripetal force at a different orbit from the Earth and finding the condition for the different orbit to have a centripetal force which would make the periods equal.

If two planets are orbiting at x_1 and x_2 where the fields are g_1 and g_2 and their velocities are v_1 and v_2 then we know that the gravitational force equals the centripetal force

$GMm/r^2=mv^2/r$, so $GM/r^2=v^2/r$, or $g=v^2/x$

$g_1=v_1^2/x_1$ and $g_2=v_2^2/x_2$.

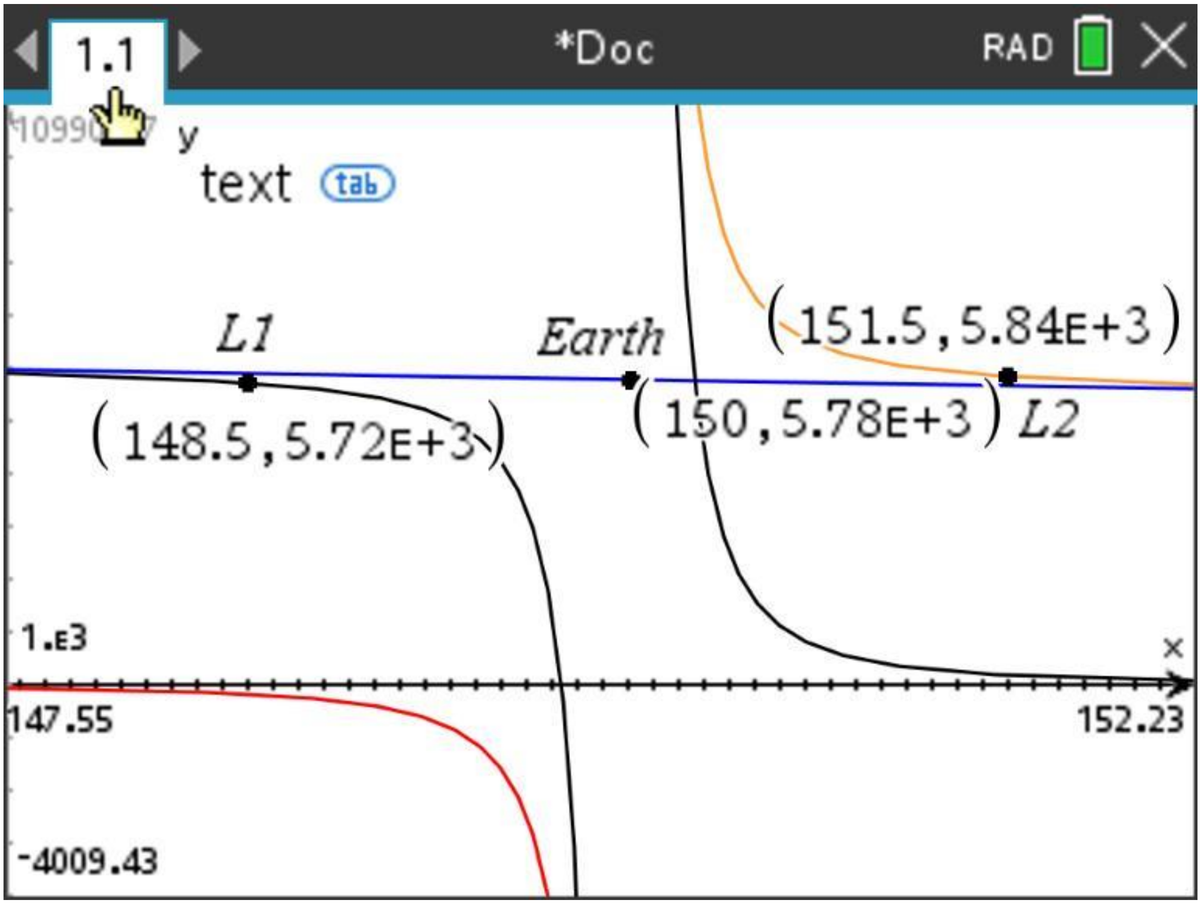
Since $v=2\pi*x/T$ where T is the orbital period, we can show that $g_1/g_2=x_1T_2^2/x_2T_1^2$ and if $T_1=T_2$ then

$g_1/g_2=x_1/x_2$

$g_2=g_1(x_2/x_1)$ (in this case, g_1 = 0.28 units and x_1 =6 units, the Sun's field at the position of the Earth in our model)

$g_2 * 21.4=x_2$ (g_2 is simply the summation of the Sun and Earth fields which when multiplied by 21.4 will give the position for an orbit of one 'year'.)

Real Data



Real data have been used here with the same colours for the gravitational force fields of Sun and Earth as in the Lagrange tns file.

Mass Sun 2×10^{30} kg

Mass Earth 6×10^{24} kg

Earth Sun distance 150×10^{9} m

You can see that things may become confused with large powers of 10. It is not easy to arrange the window to view the data hence the use of a model which provides simpler figures. In the screen above, the y values (g) are $x10^{6}$ and the x values are $/10^{9}$.

The ratios of the corresponding x and y ordinates at L1, L2 and Earth are the same , as they should be if the orbital periods are the same at each point.

Also the centripetal acceleration at Earth's orbit is $5.8 \times 10^{3}/10^{6}$ or 5.8×10^{-3} ms^{-2} which is very close to the calculated value of 5.9×10^{-3} ms^{-2}.

Ian Galloway, Copernican Revolutions, 2022.